Core Concepts and Indirect Alternatives: On the Anti-Duality of Quantifiers

Version of November 3, 2023

Abstract

This paper proposes an analysis for the long-standing puzzle observed by Chemla [2007] regarding the anti-duality of the French universal quantifier *tous*, which arises even though French has no word for 'both' to feed a *Maximize Presupposition* competition. This phenomenon has been cited as an example in language where a dual 'conceptual alternative' is at play [Buccola et al., 2018], but no formal account of it has been put forth. Furthermore, a naive implementation of the idea overgenerates anti-duality inferences in other expressions, such as *each*, *which* and *one* in English and French, which might be expected to be observed due to anti-dual counterparts in some languages like Icelandic and Japanese. We propose an account where French *tous* has an unpronounceable dual universal alternative built from a dual core concept, competition with which is licensed by the existence of a pronounceable expression equivalent in meaning, which we call 'Indirect Alternative'. This proposal accounts for *tous*'s anti-duality and lack of anti-n-ality for $n > 2$, as well as the lack of anti-duality in other quantifiers.

1 Introduction

The English universal quantifiers *all* and *every* are 'anti-dual', i.e., cannot be used if their domain is known to contain only two individuals. Instead, the dual universal quantifier *both* can be used in those contexts.

(1) a. #Lea broke all her arms.
b. #Lea broke every arm of hers.
c. Lea broke both her arms.

Percus [2006] and Sauerland [2008] argue that this is due to competition with the dual universal lexical item *both*, which, via *Maximize Presupposition* (MP) [Heim, 1991, Sauerland, 2002], makes a universal quantifier with no size restriction anti-dual.

Chemla [2007] raises a puzzle for this analysis. In French, the universal quantifier *tous* is also anti-dual. But French does not have a lexical item for 'both'. In fact, the most direct translation of 'both' is the complex definite numeral expression *les deux* ('the two').

(2) a. #Léa s'est cassé tous les bras.
Léa REFL.AUX broke all the arms
#'Léa broke all her arms.'
b. Léa s'est cassé les deux bras.
Léa REFL.AUX broke the two arms
'Léa broke both her arms.'

The French data in (2) constitute a problem for the MP-based account, for two reasons: First, MP is generally defined for individual lexical items, but *les deux*, unlike *both*, is a complex expression (and a non-constituent string). Second, if we extend it to incorporate complex expressions, it becomes a puzzle why *tous* can compete with *les deux*, but not with another identically structured numeral expression like *les trois* ('the three'), as noted by Chemla [2007]. Chemla therefore suggests that the explanation of *tous*'s anti-duality lies in the existence of a 'core concept' that can participate in competition with *tous*.

This observation has become one of the better-known examples suggesting the need for 'conceptual alternatives', that is, non-utterable meaningful objects that can compete with pronounceable linguistic material [Buccola et al., 2018]. Nevertheless, no full-fledged account of the phenomenon has yet been proposed. There is no discussion of what the relevant core concept is, and the lack of a formal account limits the applicability of the idea of conceptual alternatives because it leaves the predictions unspecified for structures beyond the ones considered in the original work. This paper takes on this challenge.

We aim to capture two empirical facts: a) the anti-duality of French *tous*, and its lack of anti-n-ality for $n > 2$, and b) the absence of anti-duality in other quantifiers, which might be expected to be observed due to anti-dual counterparts in some languages.

We propose a solution for *tous*'s anti-duality by positing the notion of *Indirect Alternative*, which relies on two main ingredients: an unpronounceable dual universal alternative, and the existence of an overt expression in the language (where *tous les*, 'all the', is replaced with *les deux*, 'the two') equivalent in meaning, which licenses competition between a *tous* expression and the unpronounceable dual alternative. Thus, *tous les* can compete with *les deux*, but only via a dual core concept, which means that it will not compete with *les trois*, because there is no trial core concept. Complexity constraints will disallow competition with dual counterparts of other expressions due to the lack of corresponding indirect alternatives.

This proposal is significant in that it maintains the intuition, dating from Gricean pragmatics, that a *pronounceable* expression be available as an alternative (albeit indirectly), while at the same time accounting for the intuition that the concept DUAL is playing a central role in the French and other duality effects as seen in (2). This proposal thus puts into question the central hypothesis of the programmatic paper Buccola et al. [2018], which argues that there can be alternatives not supported by linguistic material.

In section 2, we present the data we will aim to explain, and a summary of our solution. In section 3, we show that the anti-duality of French *tous* carries the signature of an implicated presupposition. In section 4, we present our proposal for indirect alternatives and show how it accounts for the basic facts. In 5, we discuss what our account predicts for anti-duality with other expressions, in particular *which* questions. In 6, we present some natural alternative explanations to French *tous*'s anti-duality. In 7, we conclude.

2 The puzzle and solution in a nutshell

Building on Chemla's (2007) observation, we present the puzzle in two parts that we aim to address. The first part concerns the special status of anti-duality with French *tous*. As Chemla discusses (also Buccola et al. 2018), there is no difficulty to use French *tous* or English *all* with a domain known to be n for any n greater than 2. For example, (3) might be expected to exhibit anti-triality and (4) anti-decality, but both examples are fully acceptable.

(3) Léa aime toutes les parties du triathlon.
Léa likes all the sections of.the triathlon
'Léa likes all sections of the triathlon.'

(4) Léa s'est cassé tous les doigts.
Léa REFL.AUX broken all the fingers
'Léa broke all her fingers.'

We propose a solution that pushes forward Chemla's intuition that the number concept

DUAL is special in a way that higher numbers are not. Specifically, we propose to relate Chemla's notion of core concept to the work of Harbour [2014] on number morphology. Harbour argues that universally the numerically stable categories of number morphology are the singular, dual, and plural (he recognizes in addition forms of paucal that are associated with a less stable numerical threshold). The intuition that dual is a core member of our conceptual inventory is not surprising, as it has a significant presence in human experience (and any higher number does not seem to have such a presence, in absolute terms, and relative to numbers higher than it). The special status of the dual as a core concept has allowed Harbour to explain the existence of dual morphology in grammar across otherwise unrelated languages, and will now be used to explain the seemingly unrelated phenomenon of anti-duality in universal quantification. In short, we propose that a non-lexicalized dual number concept, which we will call DUAL, is universally present across languages, and is the basis for an unpronounceable alternative to the universal quantifier *tous*, allowing MP to apply, and deriving anti-duality.

The second part of our puzzle concerns cases in which anti-duality is not observed, and might be expected to arise due to its presence in other languages. Chemla's (2007) puzzle suggests that there is a parallel between the anti-duality of *all* in languages which have a lexicalized dual counterpart *both*, and the anti-duality of *all* in languages that don't. But there are also languages such as Icelandic that have a dual counterpart of interrogative *which*, and languages such as Japanese that have a dual *which*, a dual distributive universal quantifier *each*, and a dual existential *one*. Their corresponding plural counterparts exhibit anti-duality. We show this for *which* in Icelandic ((5); XXX, personal communication) and Japanese ((6); native speaker intuition of one of the authors), and for Japanese *each* and *one* in (7) and (8) respectively.[1]

(5) a. Á hvor-um handlegg-num brotna-ði hún?
On which.DUAL-DAT arm-DAT.DEF break.INT-PST she

b. ?Á hvaða handlegg brotna-ði hún?
On which arm.DAT broke.INT-PST she
'Which arm did she break?''

(6) a. Taroo-wa dotti-no ude-o o-tta-no?
Taro-TOP IND.DUAL-GEN arm-ACC break-past-Q
'Which arm did Taro break?'

b. #Taroo-wa dono ude-o o-tta-no?
Taroo-TOP IND arm-ACC break-past-Q

(7) a. Taroo-wa dotti-no ude-mo o-tta.
Taro-TOP IND.DUAL-GEN arm-MO break-PAST
'Taro broke each.DUAL of his arm.'

b. #Taroo-wa dono ude-mo o-tta.
Taroo-TIO IND arm-MO break-PAST

(8) a. Taroo-wa dotti-ka-no ude-o o-tta.
Taro-TOP IND.DUAL-KA-GEN arm-ACC break-PAST
'Taro broke one of his arms.'

b. #Taroo-wa dono ude-ka-o o-tta.
Taro-TOP IND arm-KA-ACC break-PAST

[1] We analyze 'dotti-no … mo' as *each* because it does not allow collective or cumulative interpretations in cases where *both* does not, e.g. 'Together both/*each of my children weighs 100 kg.'

But in both English and French, no anti-duality arises in any of these cases.

(9) a. Which arm hurts you?
b. I have a problem with each arm.
c. One arm hurts me.

(10) a. Quel bras te fait mal?
which arm you cause pain
'Which arm hurts you?'
b. J'ai un problème à chaque bras.
I-have a problem to each arm
'I have a problem with each arm.'
c. Un bras me fait mal.
one arm me cause pain
'Which arm hurts you?'

On Chemla's account the absence of anti-duality with French *quel* ('which'), *chaque* ('each'), *un* ('one'), and their English counterparts requires us to stipulate that the dual counterparts of these expressions are not core concepts in contrast to dual *all*. But Chemla's account is not equipped to explain the difference between cases like *all* where anti-duality extends to languages that lack a relevant dual expression and cases like *which*, *each* and *one* where it does not.

Furthermore, one may wonder specifically about the differences between universal quantifiers, where *all*, *every*, and *tous* are anti-dual, but not *each* or definite plurals, the latter shown in (11) and (12) for English and French respectively.

(11) My arms hurt.

(12) Mes bras me font mal.
my arm me cause pain
'My arms hurt.'

An account that simply has the dual as a core concept would struggle to explain the lack of anti-duality in these cases. Our proposal however provides a natural explanation for these cases. In a nutshell, our solution will be to limit the ability of the core concept DUAL to compete with structural alternatives like French *tous* to those cases where the core concept can be 'linked' – in a way to be made precise – to an existing pronounceable expression. For French *tous les* ('all the'), the mechanism links it to *les deux* ('the two').

We will call such an alternative an *indirect alternative*. In general, indirect alternatives are expressions which are not directly generated as an alternative via known alternative-generation algorithms (e.g., standard structural alternatives as proposed in Katzir 2007), but nevertheless enter competition because they are equivalent in meaning to an unpronounceable, conceptual alternative such as the DUAL core concept. In this proposal, the ability of *tous* to compete with *les deux* necessarily depends on the presence of the dual core concept, which explains why anti-duality but not anti-triality is observed with French *tous*, as there is no trial core concept.

Furthermore, indirect alternatives will be subject to a familiar complexity limit on alternatives, in that they cannot surpass the complexity of the expression they are the competitor to. This requirement will reliably block other expressions seen in (9) and (10) from having as indirect alternatives *which of the two*, *one of the two* and *each of the two* or their French counterparts.

3 *Tous*'s anti-duality is an implicated presupposition

In this section, we summarize the proposal of the anti-duality of English *all* as an implicated presupposition as presented in Percus [2006] and Sauerland [2008]. In particular, we will show arguments from Sauerland [2008] where anti-duality carries the signature of an implicated presupposition, namely in its epistemic status and (crucially) its projection properties under universal quantifiers, which makes it incompatible with a mere lexical specification. We show that the facts presented for English *all* carry over to French *tous*.

The data below illustrate that the use of *all* is odd when the referents are known to be 1, as in (13), or 2, as in (14), contrasting with a situation where the referents are known to be more than 2, as in (15).

(13) a. #Billy broke all his noses.
b. #Billy broke every nose of his.
c. #Billy s'est cassé tous les nez.
Billy textscrefl.AUX broke all the noses

(14) a. #Billy broke all his legs.
b. #Billy broke every leg of his.
c. #Billy s'est cassé toutes les jambes.
Billy REFL.AUX broke all the legs

(15) a. Billy broke all his fingers.
b. Billy broke every finger of his.
c. Billy s'est cassé tous les doigts.
Billy REFL.AUX broke all the fingers

The English examples are said to be explained by the principle in (16), first proposed in Heim [1991]; the relevant competitor ψ is *both*, for anti-duality:

(16) ***Maximize Presupposition***
Do not use ϕ in context c if ψ is an alternative to ϕ such that:
a. ψ has a stronger presupposition than ϕ.
b. ϕ and ψ are contextually equivalent in c.

Maximize Presupposition states that, given two alternatives with an identical at-issue content, a speaker should use the alternative that presupposes more. To illustrate, *all* and *both* have the same at-issue content: they state that a property in their scope is true of all elements in their restriction. Comparing the presuppositions of *all* and *both*, *both* presupposes that there are two individuals, while *all* presupposes nothing. Therefore, if there are two individuals in the context of which a property is true, one must use *both*, making *all* infelicitous in such a context. This explains the contrast between (14) and (15), because legs generally come in twos, but not fingers. It also explains why *all* carries the *implicated presupposition* that it is not the case that the relevant domain consists of exactly two entities.

English *all*'s anti-duality is straightforwardly explained by MP in its competition with *both*. How about French *tous*'s anti-duality? If French lacks a word for *both*, as mentioned in the introduction, we might expect its anti-duality to not arise as an implicated presupposition, but instead to be lexically encoded in the meaning of *tous*. We argue against this possibility, and show that the anti-duality of French *tous* exhibits the behavior of an implicated presupposition, in parallel with English *all*. We focus on two signature characteristics from implicated presuppositions [Sauerland, 2008]: a weak epistemic status, and

the lack of universal projection in the scope of a universal quantifier.

First, (17) shows that the anti-dual inference of *every* and *tous* is epistemically weak: if there is ignorance about whether the domain contains two individuals, these quantifiers can be used (the French data here and throughout the paper is from two native speakers).

(17) *Context: I don't know how many students there will be in my next class, there could be 2.*

a. Every student in my next class will have to work hard.

b. Dans mon prochain cours, tous mes étudiants vont devoir travailler dur.
in my next class all my students will have-to work hard

The second characteristic feature of implicated presuppositions is that they need not project universally in the scope of a universal quantifier, in contrast to a typical presupposition [Chemla, 2009, Heim, 1983, Schlenker, 2008, a.o.]. Again, we see that French *tous* patterns with English *all* in this respect:

(18) *Context: Several candidates applied. Some have written only one paper, others two, and the rest have written more than two.*

a. I checked, every candidate sent every paper of theirs.

b. J'ai vérifié, chaque candidat a envoyé tous ses articles.
I.AUX checked each candidate AUX sent all their articles

Contrast this with (19), where the dual presupposition of *both* and the uniqueness presupposition of the possessive pronoun *son* ('their') project universally:

(19) *Context: Several candidates applied. Some have written only one paper, others two, and the rest have written more than two.*

a. #Every candidate sent both papers of theirs.
⇝ *every candidate has exactly two papers*

b. #Chaque candidat a envoyé son article.
each candidate AUX sent their article
⇝ *every candidate has exactly one paper*

Note that the weak epistemic status of *tous*'s anti-duality does not in principle rule out the possibility of it being lexically encoded, namely as an epistemically weak presupposition. However, the projection facts rule out lexical specification: if *tous*'s anti-duality were a presupposition, we should expect it to behave like one and project universally from the scope of a universal quantifier.

In sum, the facts above strongly suggest that the anti-dual meaning component of French *tous* is indeed an implicated presupposition.

4 Proposal: Indirect Alternatives

We have put aside the possibility that the anti-duality of French *tous* is lexically encoded; we have furthermore established that it carries the signature of an implicated presupposition. Our proposal will therefore maintain that the anti-duality of *tous* is an implicated presupposition. However, since French does not have a word for 'both', we need to find a competitor to *tous* which carries a dual presupposition. Once this competitor is found, the use of *tous* is predicted, via Heim's MP in (16), to implicate anti-duality, as observed.

We propose that the definite dual expression *les deux NP VP* ('the two NP VP') is

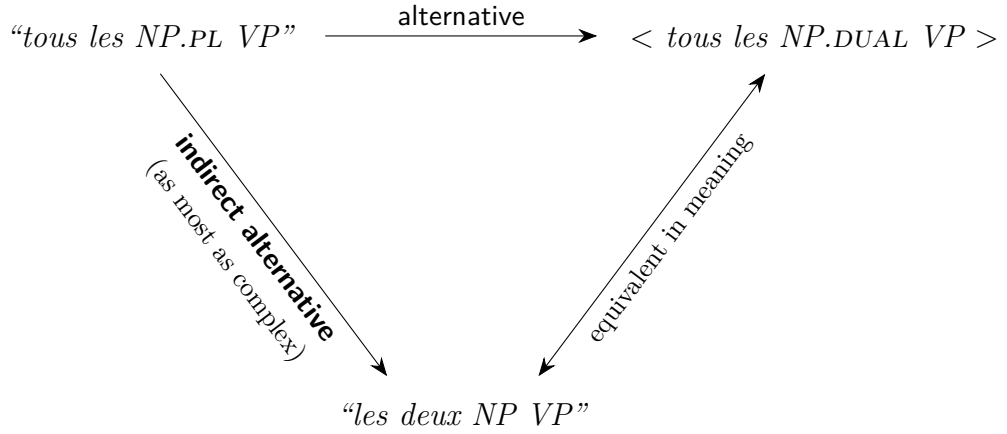


Figure 1: Indirect competition with *tous les NP VP*

an alternative to *tous les NP VP* ('all the NP VP'), albeit an atypical one, which we call an *indirect alternative*, as illustrated in Figure 1. It's an alternative that is not generated directly by the grammar's basic alternative generation mechanism, but that is equivalent in meaning to a directly generated, but unpronounceable, alternative (written between < > in Figure 1). In this case, we posit that in French, a *tous* expression can generate as an alternative a dual universal expression (with meaning 'both') that is not pronounceable. This will therefore allow for *les deux* to enter in competition with the universal expression, but not *les trois* (and so on) because there is no corresponding trial universal expression.

In addition, there will be a complexity limit on the indirect alternative. *Les deux NP VP* is not more complex than *tous les NP VP*, which means that it can act as an alternative to it. This complexity restriction will appropriately predict no anti-duality inferences for expressions other than the universal quantifier. For instance, *which of the two* is more complex than *which*, and therefore cannot act as an alternative to it (furthermore, there is no other expression in English equivalent in meaning to *which of the two* that is less complex than it).

We first show in section 4.1 that if *les deux NP VP* is generated as an alternative, then MP is licensed, because it is contextually equivalent to *tous les NP VP*. In the following two sections we show that it can be generated as an indirect alternative. In 4.2, we present the notion of indirect alternative, and in 4.3 show that *les deux NP VP* can be one to *tous les NP VP* as long as there is an equivalent unpronounceable alternative to *tous les NP VP*. In section 4.4, we define the nature of the unpronounceable dual universal expression. In section 4.5, we discuss how an immediate prediction of the proposal is borne out. In section 4.6, we offer a translation of this solution in a Meaning First framework, which provides a natural way to account for core concepts.

4.1 If 'the two' is generated as an alternative, then MP is licensed

First, we show that a sentence containing *les deux NP* and one containing *tous les NP* in the same position are contextually equivalent, and only differ in their presupposed content, thus allowing for MP to apply.

(20) a. Tous les verres sont pleins.
all the cups are full
'All the cups are full.'

b. Les deux verres sont pleins.
the two cups are full
'The two cups are full.'

We start with the definite numeral expression in (20-b). Numerals are typically assumed to indicate a lower bound (e.g., 'at least 2'), and achieve an exact number reading (e.g., 'exactly 2') through pragmatic reasoning. We will skip this pragmatic step irrelevant for our purposes (and unaffecting our results, we believe), and shortcut to *deux* ('two') having the meaning 'exactly 2'. Next, we adopt the familiar view of the definite article to be a maximality operator [Sharvy, 1980], taking the maximal sum of individuals present in the extension of its argument.[2] We take the extension of a predicate to contain both atoms and pluralities, and denote the maximality operator using the σ operator, as in (21-b).

(21) a. $[\![\text{ deux }]\!] = \lambda P.\lambda x.P(x) \wedge |\{y : atom(y) \wedge y \sqsubset x\}| = 2$
b. $[\![\text{ les deux verres }]\!] = \sigma x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2]$

We now make this compose with the VP *sont pleins* ('are full'). Since *plein* ('full') is a distributive predicate, it must compose with the plural individual via a distributivity component. We remain agnostic as to where the distributivity component is introduced, whether by a separate operator or in the meaning of the predicate. In (22-a), we give it as part of the meaning of *plein*.

(22) a. $[\![\text{ sont pleins }]\!] = \lambda x.\forall z.atom(z) \wedge z \sqsubset x \rightarrow full(z)$
b. $[\![\text{ les deux verres sont pleins }]\!] =$
$\forall z.atom(z) \wedge z \sqsubset \sigma x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2] \rightarrow full(z)$

Now we turn to the semantics of the universal expression. We take French *tous* to be a universal quantifier over subparts of a definite plurality. This semantics is based on the selectional properties of *tous*, which can only compose with a noun phrase if it is plural-marked, like in English, and via a definite article, unlike in English. *Tous*-phrases are compatible with collective readings,[3] which suggests that the universal quantification is not restricted to atoms. This entails that a distributivity component must be included in the semantics in this case as well (again, for simplicity, we introduce it at the level of the verbal predicate, as in (22-a)).

(23) a. $[\![\text{ tous }]\!] = \lambda x.\lambda Q.\forall z.z \sqsubset x \rightarrow Q(z)$
b. $[\![\text{ les verres }]\!] = \sigma x.cup(x)$
c. $[\![\text{ tous les verres }]\!] = \lambda Q.\forall z.z \sqsubset [\sigma x.cup(x)] \rightarrow Q(z)$
d. $[\![\text{ tous les verres sont pleins }]\!]$
$= \forall z.z \sqsubset [\sigma x.cup(x)] \rightarrow [\forall y.atom(y) \wedge y \sqsubset z \rightarrow full(y)]$
$\equiv \forall z.atom(z) \wedge z \sqsubset [\sigma x.cup(x)] \rightarrow full(z)$

The at-issue meanings in (22-b) and (23-d) are equivalent. (22-b) states that every atomic subpart of the maximal sum that makes *cup* true and has exactly 2 atoms is full. Ignoring the presuppositional component, the assertive component simply states that all cup atoms are full. This is equivalent to the meaning of (23-d).

[2] As far as we can see, our analysis is equally compatible with the maximal informativity based analysis of the definite determiner of von Fintel et al. [2014], but the empirical disadvantages of Sharvy's analysis compared to the maximal informativity based one do not cause any problems for the data we are considering.

[3] Just like *all*, e.g. 'All the ants lifted the piano.'

The presuppositional component of (23-d), introduced by the σ operator, is that there is a maximal sum of cups. In (22-b), it is that there is a maximal sum of cups which contains exactly two atoms. Now we apply MP, which means that if the number of atoms is known to be exactly two, the definite description that presupposes that there are exactly two cups is felicitous, and therefore we cannot use the *tous* expression, which presupposes less.

One detail we have so far ignored is the difference in meaning typically observed between quantificational expressions and definite plural expressions. While plural definites are interpreted as 'quasi-universal', they are associated with two properties that distinguish them from universal quantification: non-maximality, namely exception tolerance, and homogeneity, referring to an apparent wide scope reading with respect to negation.

(24) a. **Non-maximality:**
J'ai lu les livres sur la liste.
I read the books on the list.
$\rightsquigarrow$ *compatible with not reading one or two books on the list.*

b. **Homogeneity:**
Je n'ai pas lu les livres sur la liste.
I didn't read the books on the list.
$\rightsquigarrow$ *I read no or almost no books on the list.*

A universal quantifier which contains a plural definite, like English *all* and French *tous*, is known to erase these two properties [Malamud, 2012, Križ, 2015]: in its presence, neither homogeneity nor non-maximality are observed.

(25) a. **Non-maximality removal by *tous/all*:**
J'ai lu tous les livres sur la liste.
I read all the books on the list.
$\rightsquigarrow$ *not compatible with not reading one or two books on the list.*

b. **Homogeneity removal by *tous/all*:**
Je n'ai pas lu tous les livres sur la liste.
I didn't read all the books on the list.
$\rightsquigarrow$ *I read some books on the list.*

In the semantics provided above, these properties are ignored. There have been several proposals to capture the semantics of homogeneity and non-maximality [Krifka, 1996, Malamud, 2012, Križ, 2015, Križ and Spector, 2021], some of which discuss the difference between the semantic contribution of *tous*, which removes homogeneity and non-maximality, and that of the covert distributivity operator, which doesn't [Križ, 2015, Križ and Spector, 2021]. While some of these proposals allow for contextual equivalence of the universal quantifier expression and the definite plural expression, showing this would require importing the particularly heavy machinery involved in the formal implementations of those proposals, which we will not do here. Doing so would also not explain the whole range of facts, in particular, when a universal quantifier scopes below negation, and anti-duality persists, which requires a competitor that also lacks homogeneity (see (27), which we address in upcoming discussion).

However, our goal is not to compare a quantificational expression with a plain plural definite, but with a plural definite containing a numeral. Such expressions have not received much attention in the literature, where, as far as we know, the only mention is in Križ [2015]. Križ reports that the presence of a numeral appears to remove non-

maximality, as shown in (26-a), but he tentatively claims that it doesn't remove homogeneity in English. However, he reports a possible contrast with French, where definite numerals don't seem to conserve their homogeneity. Our own judgments might also reveal a contrast with the two languages, although the data is very subtle, and it seems like there is at least one parse in both languages compatible with homogeneity removal (with the role of prosody unclear).

(26) a. **Non-maximality removal by a numeral:**
J'ai lu les deux/dix livres sur la liste.
I read the two/ten books on the list.
$\rightsquigarrow$ *not compatible with not reading one or two books on the list.*

b. **Homogeneity removal by a numeral?**
Je n'ai pas lu les deux/dix livres sur la liste. ✓J'en ai lu que un/cinq.
I didn't read the two/ten books on the list. ?✓I only read one/five.
$\rightsquigarrow$ *compatible with reading some books on the list.*

The facts with non-maximality removal appear to be quite robust, and should be enough to make the claim that a definite numeral expression is contextually equivalent to a quantified expression, when these are unembedded. As for homogeneity removal, we will rely on the French facts, which are those we need to make our point. There is a clear contrast between a plain definite plural and one with a numeral, where the one with a numeral has a non-homogeneous reading ($\neg > N$) available. This homogeneity removal, however, appears less robust than the one with *tous*, in that an apparent wide scope reading of the numeral ($N > \neg$) still appears to be available. One could argue that homogeneity removal with *tous* is particularly clear because of the presence of a scalar implicature (due to *tous* competing with *quelques* ('some')) incompatible with the homogeneous reading, which the definite numeral lacks. Either way, we would like to emphasize the crucial point for our purposes, namely that there exists a non-homogeneous parse of the definite numeral. This matters in a sentence where a quantifier expression like *tous les verres* scopes below negation, as in (27), and still has an anti-duality inference. This inference would rest on the non-homogeneous parse of the definite numeral expression *les deux verres* scoping below negation, as in (28).

(27) a. Pas tous les verres sont pleins.
not all the cups are full
'Not all the cups are full.' *odd if only 2 cups*

b. Les deux verres ne sont pas pleins.
the two cups neg are neg full
'The two cups are not full.' ✓$\neg > 2$

4.2 Indirect alternatives: definitions

In this section we define the notion of *indirect alternative*, that depends on the existence of *unpronounceable alternatives*. We leave the possibilities of what counts as an unpronounceable alternative to the next section, and in this section simply assume that it exists.

We adopt the standard alternative generation mechanism proposed by Katzir [2007], where alternatives are obtained from deleting constituents, or replacing a constituent with a lexical item of the same syntactic category.

(28) A **structural alternative** of a parse tree ϕ is a parse tree ψ obtained from ϕ

by deleting constituents, or replacing constituents with lexical items of the same syntactic category. [Katzir, 2007]

The notion 'of the same syntactic category' is crucial for our purposes.[4] This restriction entails that *les n NP*, for any numeral n, is not generated as an alternative to *tous les NP* because *tous*, a quantifier, is not replaceable by *les*, a determiner, and *les* is not replaceable by a numeral. So *tous les NP* cannot have *les n NP*, for any n, as an alternative. Thus, *les deux NP* is not generated as a direct alternative via the Katzirian alternative generation algorithm.

Instead, we propose that we can generate it as an indirect alternative via an unpronounceable alternative. An unpronounceable alternative, defined in (29), is a linguistic object directly generated as an alternative that is unpronounceable for independent reasons such as the lack of a lexical item in the language.

(29) An **unpronounceable alternative** is an expression obtained from the grammar's alternative generation mechanism, but cannot be pronounced.

We assume that direct competition with an unpronounceable alternative is impossible. However, competition is licensed with a pronounceable expression equivalent in meaning to it, an indirect alternative. We define the notion of indirect alternative below.

(30) A pronounceable parse tree I is an **indirect alternative** of a parse tree S iff. there is an unpronounceable alternative S_X of S such that:
(i) $[\![I]\!] \equiv [\![S_X]\!]$, and
(ii) $I \preccurlyeq S$ $:=$ I has at most as many nodes as S.

Both the Katzirian alternative and the indirect alternative is defined to be at most as complex, in some way, as the expression it is an alternative to. However, the notions of complexity used in the two definitions are not the same. Katzir's notion of complexity, which is directly integrated into the definition of structural alternatives, only allows comparison between parse trees that are related by the Katzirian algorithm. This means that an expression cannot be compared to an indirect alternative, which is by definition not related in a Katzirian way. Thus we appeal to a more general notion of complexity, which is number of nodes in the tree, which is applicable to comparing any two parse trees.

4.3 'The two' satisfies the complexity limit for indirect alternatives

Now we show that *les deux NP VP* is a good candidate for being an indirect alternative to *tous les NP VP*, because it is at most as complex as it, i.e., has at most as many nodes as it. *Les deux NP* and *tous les NP* have identical complexity, with three overt terminal nodes each (each a syntactic head). Then they both merge in identical fashion with the VP. (We are ignoring possible additional heads, which, if present, should be present in both structures, thus not affecting relative complexity)

(31) a. $[_{QP}\ [_{Q}$ tous $]\ [_{DP}\quad [_{D}\quad$ les $]\quad [_{NP}$ verres $]]]\ [_{VP}$ sont pleins$]$
b. $[_{DP}\ [_{D}$ les $]\quad [_{NumP}\ [_{Num}$ deux $]\ [_{NP}$ verres $]]]\ [_{VP}$ sont pleins$]$

Thus, the complexity requirement for *les deux NP VP* being an indirect alternative to *tous les NP VP* is satisfied. This means that it is an indirect alternative to it if there is

[4] As far as we can tell, this notion was in fact not crucial to explain the data originally used to develop Katzirian alternatives.

an unpronounceable expression that is directly generated as an alternative to *tous les NP VP*, and equivalent in meaning to *les deux NP VP*. The nature of this unpronounceable alternative is the topic of the upcoming section 4.4.

This complexity requirement accurately blocks other NP-containing expressions from having indirect dual alternatives. For instance, there is no expression in English or French equivalent to 'which of the two NP', simpler than 'which (of the) NP'. So, even if 'which of the two NP' is equivalent in meaning to an unpronounceable dual alternative, there is no indirect alternative licensing competition, and no anti-duality is derived. The same reasoning applies to other expressions that do not exhibit anti-duality such as 'each NP', 'one NP', 'the NP'. We continue the discussion of anti-duality, or lack thereof, with other quantifiers and cross-linguistically, in section 5.

4.4 The unpronounceable dual alternative to *tous les NP*

The data on French *tous*'s anti-duality reveals that language makes the concept 'two' important in some sense. This observation corroborates data in various domains in which duality is lexicalized, but not triality, etc. For instance, *both* is lexicalized in English, but not something equivalent to 'all the 3'. Languages have dual pronouns, but many fewer have trial pronouns. Thus, there is on the one hand an intuition that the number '2' is conceptually more prominent in some way than any other numeral, and on the other this prominence is grammaticalized, as there are reflexes in language that show this. Following Chemla's (2007) insight, we propose that a universal dual number concept DUAL is integrated in the grammar of every language, and that this core concept is responsible for the presence of a dual universal alternative to the French universal quantifier *tous*.

In this section, we propose the existence of a dual universal expression *tous les* DUAL *NP*, built from a core number concept DUAL, to be an unpronounceable alternative to *tous les* PL *NP*, which licenses competition with the indirect alternative *les deux NP*.

4.4.1 The core concept DUAL

We propose that there exist *core concepts*, which include the core concept DUAL, which are universally present in the lexicon of any language. We assume that core concepts are either phonologically null or non-null, and in the latter case they can be phonologically realized together with another operator. For example, in English, we assume that DUAL in the restrictor of a universal quantifier is realized as *both*. In French, DUAL has no phonological realization in the scope of a universal quantifier.

We assume that the core concept DUAL is available alongside the number concepts plural and singular, and can thus combine with any NP. As a consequence, in a language where dual is unpronounced, a string containing a plural-marked NP is ambiguous between a plural and a dual interpretation. This will be possible in the scope of a universal quantifier as well, allowing the structure *tous les* DUAL *NP* to be generated.

This proposal can be aligned with that of Harbour [2014], who proposes that the dual arises from the interaction of primitive number features [−atomic] and [+minimal]. We depart from Harbour and subsequent authors in assuming that the dual is universally present, meaning that both [atomic] and [minimal] features are present in all languages, despite not being morphologically expressed. Their universality is thus often invisible, but is revealed in some corners of grammar, such as with the anti-duality of *tous* in French. See the appendix (section 8) for an implementation of the following proposal using Harbour's features. In this section, we simply assume that universal number features include SG, PL and DUAL.

4.4.2 Blocking the dual universal expression

We propose an economy principle that blocks this dual universal structure from pronunciation because of the presence of an unambiguous string equivalent in meaning to it (once it combines with a VP), and at most as complex as it: *les deux NP*. This principle, which we call 'Avoid Ambiguity', defined in (32), encodes the preference to express a given meaning using an unambiguous string (compatible with only one logical form) rather than an ambiguous string (compatible with more than one logical form), as long as the unambiguous string is at most as complex as the ambiguous string.

(32) **Avoid Ambiguity:** if a string S is ambiguous between two parses P1 and P2, and there is a string S' with only one parse, whose meaning is equivalent to P1, and which is structurally at most as complex as S, then P1 is blocked from being pronounced.

The string 'tous les NP' is ambiguous between a plural reading *tous les* PL *NP* and a dual reading *tous les* DUAL *NP*. *Tous les* DUAL *NP VP* is equivalent in meaning with the expression *les deux NP VP* (as we are about to show), which is not more complex than it. Therefore, according to (32), the parse *tous les* DUAL *NP VP* is blocked from pronunciation.

We now show the meaning equivalence between *tous les* DUAL *NP VP* and *les deux NP VP*. We propose that the number concept DUAL has the semantics equivalent to 'exactly 2'.[5]

(33) a. $[\![\text{DUAL}]\!] = \lambda P.\lambda x.P(x) \wedge |\{y : atom(y) \wedge y \sqsubset x\}| = 2$
b. $[\![\text{les DUAL verres}]\!] = \sigma x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2]$
c. $[\![\text{tous les DUAL verres}]\!] =$
$\lambda Q.\forall z.z \sqsubset [\sigma x.cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2] \rightarrow Q(z)$

We compose this dual universal quantifier with the predicate *sont pleins* ('are full') (whose semantics, as assumed earlier in (22-a), contains a distributivity component).

(34) $[\![\text{tous les DUAL verres sont pleins}]\!] =$
$\forall z.z \sqsubset [\sigma x.cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2] \rightarrow [\forall y.atom(y) \wedge y \sqsubset z \rightarrow full(y)]$
$\equiv \forall z.atom(z) \wedge z \sqsubset [\sigma x.cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2] \rightarrow full(z)$

We can see that this meaning is equivalent to the one generated by the definite numeral expression, which we already derived in (22-b), repeated below (modulo interaction with homogeneity, which we decided should not play a role in this case, as discussed in that section).

(35) $[\![\text{les deux verres sont pleins}]\!] =$
$\forall z.atom(z) \wedge z \sqsubset \sigma x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubset x\}| = 2] \rightarrow full(z)$

4.4.3 Indirect competition licensed

Tous les DUAL *NP VP* is a Katzirian alternative to *tous les* PL *NP VP*, where PL is replaced by DUAL. However, as we showed above, *tous les* DUAL *NP VP* is blocked from

[5]The result would have been identical, in this case, if DUAL had had a lower bound semantics, as is generally proposed for numerals. However, we assume here an exact semantics for DUAL, in order to align it with the proposal in the next section, in which we reduce DUAL to a feature bundle whose semantics will be 'exactly 2'.

pronunciation. Therefore competition with it is not licensed, but competition with the indirect alternative *les deux NP VP* is, since it is equivalent in meaning to *tous les* DUAL *NP VP*, as shown above, and at most as complex as *tous les NP VP*.

As a result, the anti-duality of French *tous les NP* ('all the NP') comes from MP in the indirect competition with *les deux NP* ('the two NP') via the conceptual alternative *tous les* DUAL *NP* (after combining with the rest of the sentence).

We have given a solution that relies on the existence of core concepts, which, even when blocked from pronunciation, can play a role in alternative generation, if an indirect alternative is present.

4.5 A borne out prediction: *tous les deux*

Based on the current assumptions, the analysis makes a prediction: the numeral *deux* ('two') can be in the restrictor of *tous*.

Recall that the dual universal alternative 'tous les DUAL NP' is blocked due to its phonological identity with a structure with different meaning, namely, 'tous les NP', and the availability of a semantically equivalent unambiguous structure, namely, 'les deux NP'.

Now, we can also have a structure 'tous les DUAL deux enfants'. The spellout of this structure is unambiguous. Therefore it shouldn't be blocked, and we thus predict the numeral to be in the restrictor of *tous*.

This is what we observe, in some cases, as shown below.

(36) Tous les deux sont venus.
all the two are came
'Both came.'

Note this is only observed when the noun is elided. When it is not, however, no numeral is possible.

(37) a. *Tous les deux enfants sont venus.
all the two children are came
b. *Tous les trois/dix enfants sont venus.
all the three/ten children are came

The fact that *deux* is not possible when the noun is not elided is thus due to an independent constraint on numerals being blocked in the scope of *two*. The reason behind this intriguing phenomenon is beyond the scope of this paper.

4.6 Indirect alternatives in the Meaning First framework

In this section, we maintain the idea of competing with a core concept, but switch to a Meaning First framework [Sauerland and Alexiadou, 2020], in which such objects can be naturally incorporated into the ontology.

The Meaning First framework contrasts with a standard syntax-first Y-model in postulating that language results from the compression of structured thoughts into phonological representations as shown in Figure 2, where thought structures are universal, and recoverable pieces of thought need not be phonologically realized.

Figure 2: Meaning First model of grammar [Sauerland and Alexiadou, 2020]

A Meaning First architecture relies on the necessity to postulate some universal primitive concepts from which conceptual representations are built. We can thus say that the 'core concept' DUAL is one of those, and can freely combine with other concepts as part of the combinatorial system of conceptual objects that feeds language.

We assume that alternatives are generated at the thought level, but that they can only feed into competition mechanisms under some specific conditions.

(38) **Thought competition:** A thought T will compete with an alternative thought T' if there exists a compressed form (phonological form) of a thought $C(T'')$ such that $C(T'') \preccurlyeq C(T)$ (under some notion of complexity adapted to compressed forms), and $T'' \equiv T'$.

In addition, we can stipulate that a thought structure always has as an alternative a thought structure that differs from it by a primitive concept (either added, removed, or replaced with another). Let T be a thought of a universal quantificational claim over subparts of a plurality, which would be compressed into 'all the Ps Q'. T^* is an alternative to T where the core concept DUAL was added (via predicate modification) to its restrictor, where DUAL is a property of pluralities which counts its atoms and returns true if there are exactly 2.

(39) a. $T = \forall y \sqsubset \sigma x \,.\, P(x) \rightarrow Q(y)$
b. $T^* = \forall y \sqsubset \sigma x \,.\, \text{DUAL}(x) \wedge P(x) \rightarrow Q(y)$

As shown in section 4.4.1, T^* is equivalent to the meaning of *les deux Ps Q* (i.e., to the thought which compresses into this phonological form). Therefore competition between T and T^* is licensed.

5 Anti-Duality and its Absence with Other Quantifiers

In this section, we further discuss the second one of the puzzles for Chemla's (2007) proposal that we mentioned in the introduction.

The puzzle is that languages like Icelandic and Japanese express duality with quantifiers other than *all*—namely *some*, *which*, and *each*—where neither English nor French express duality. We furthermore observe in the languages that express duality a corresponding anti-duality with the quantifiers that do not express duality. But in English and French, which do not express duality with *some*, *which*, and *each*, anti-duality is not observed. We initially discuss *which*-phrases and will return to other quantifiers later in this section. Recall from above that, in both Icelandic (40) and Japanese (41), the dual marked *which*-phrase in a. must be used when the domain of the *which*-phrase has exactly two elements. The number-general *which*-phrase in b. can in both languages only be used if the domain of the *which*-phrase has three or more elements.

(40) ICELANDIC, repeated from (5)
a. Á hvor-um handlegg-num brotna-ði hún?
on which.DUAL-DAT arm-DAT.DEF break.INT-PST she
'Which arm did she break?'
b. ?Á hvaða handlegg brotna-ði hún?
on which arm.DAT broke.INT-PST she

(41) JAPANESE, repeated from (6)

a. Taroo-wa dotti-no ude-o o-tta-no?
Taro-TOP IND.DUAL-ACC arm-NOM break-past-Q
'Which arm did Taro break?'
b. #Taroo-wa dono ude-o o-tta-no?
Taroo-TOP IND arm-ACC break-past-Q

The data points in (42) and (43) show that the number-general form of *which* must be used with domains of numerosity greater than two in both Icelandic and Japanese.

(42) ICELANDIC
a. *Á hvor-um fingr-i brotna-ði hún?
on which.DUAL-DAT finger-DAT broke.INT-PST she
b. Á hvaða fingr-i brotna-ði hún?
on which.PL finger-dat broke.INT-PST she
'Which finger did she break?'

(43) JAPANESE
a. #Taroo-no dotti-no yubi-ga oreta-no?
Taro-GEN IND-DUAL-GEN finger-NOM broke-Q
b. Taroo-no dono yubi-ga oreta-no?
Taro-GEN IND finger-NOM broke-Q
'Which of Taro's fingers broke?'

The number-general *which*-phrases in English and French, on the other hand, can be used also if the domain of the *which*-phrase has two elements as shown by (44) and (45).

(44) ENGLISH, repeated from (9)
Which arm hurts you?

(45) FRENCH, repeated from (10)
Quel bras te fait mal?
which arm you cause pain
'Which arm hurts you?'

The absence of anti-duality in English and French is not predicted by a proposal that simply proposes that DUAL is a core concept. Such an account could capture the difference between the anti-duality of French *tous* ('all') and the absence of anti-duality of English *which* and French *quel* only by means of the stipulation that the meaning of English *both* amounts to a core concept, while the meaning of Icelandic *hvor* ('which.DUAL') and Japanese *dotti* ('IND.DUAL') are not. In this section, we show how the account we proposed in the previous section predicts the absence of anti-duality for English and French *which*-phrases and other relevant cases.

To verify that our account predicts the absence of anti-duality for English (44), we first note that modern English unlike Icelandic and Japanese lacks a dual counterpart of *which*. Furthermore, we need to check that there is no indirect alternative in the sense we defined in (30) in English that is equivalent to the unpronounceable structure where DUAL combines with *which*. But there are at least two meanings one might ascribe to such a structure, namely, duality could apply to either the domain of a quantifier or to the verifier thereof. We define these two interpretations of number marking on a quantifier formally in (46), where we assume that a morpheme M with an interpretation M of type $\langle e, t\rangle$ is a number morpheme if $\forall x, y \in D_e \,.\, \#x = \#y \rightarrow M(x) = M(y)$.

(46) For a number-morpheme M occurring with a quantificational noun phrase in a structure T = '[Q NP]-M S', we distinguish:

a. *domain application* of M: The interpretation of T is equivalent to the application of M to the referent of 'the R' conjoined with the interpretation of 'Q(R)(S)'.

b. *verifier application* of M: The interpretation of T is equivalent to the application of M to the referent of 'the [R and S]' conjoined with the interpretation of 'Q(R)(S)'.

First, consider domain duality. Both Icelandic *hvor* and Japanese *dotti* exhibit domain duality. This follows from the datum in (5) and (6) as the domain of *which* is the set of two arms, but the expected answer is about a singular arm. Why do we not observe domain plurality with *which*-questions in English? In English, one way to express domain duality in a question is to use partitive *of* and the numeral *two* as in (47).

(47) Which of the two arms hurts you?

But (47) is not an indirect alternative of (44) because it is more complex. The same holds for any other way of expressing a dual meaning equivalent to (47) that we can think of: 'Which of your left and right arm hurts you?', 'Does your left or right arm hurt you?' and 'Which arm of two hurts you?'. Therefore, our concept of indirect alternative predicts for domain duality no implicated presupposition should arise in English and French.

Let us now discuss verifier duality. With plural marking, verifier plurality is attested with plural marking on interrogative pronouns in several languages including English, Farsi, Spanish, Hungarian, Greek, and German [Maldonado, 2020, Elliott et al., 2022, Alonso-Ovalle and Rouillard, 2023]. Consider the English plural *which*-phrase in (48). Domain plurality would be satisfied if the addressee has multiple fingernails. But the effect of plurality in (48) is stronger – it is interpreted as a presupposition of the question that the addressee painted a plurality of their fingernails.

(48) Which fingernails of yours did you paint?

The interpretation of dual marking in Icelandic (5) and Japanese (6) – it only applies to the domain and the presupposed answer is singular. Regarding the underlying cause of the difference, one initial hypothesis could be that it arise from the morphological difference between English *which*, that doesn't bear number marking, and Icelandic *hvor* ('who'-DL), which does. But as Maldonado [2020], Elliott et al. [2022], Alonso-Ovalle and Rouillard [2023] discuss, plurality is interpreted as a verifier presupposition even in languages where a singular and plural form of *who* are both available such as Spanish *quien* ('who'-SG) and *quien-es* ('who'-PL).

At the same time, we observe that in all cases of dual marking that we have discussed, the duality presupposition applies to the domain. For the case of English *both* the negated example (49) demonstrates the domain application. We observe that if Lea has ten fingernails, (49) results in a presupposition failure, even if she painted exactly two of them. But with the verifier application of the duality presupposition, this would not be predicted because the fingernails of hers that Lea painted would then indeed be two and she furthermore didn't paint all of her fingernails.

(49) #Lea didn't paint both fingernails of hers.

There are also languages that have dual marking that applies to the verifier instead

of the domain, as it does in Icelandic and Japanese. This is the case for Slovenian, which has dual marking on adjectives and nouns.

(50) Kater-a računalnik-a sta pokvarjen-a?
which-DUAL computer-DUAL are.DUAL broken-DUAL?
'Which two computers are broken?'

Although this meaning is available in Slovenian, no anti-duality is predicted in counterparts with non-dual marking languages because no indirect alternative exists. Two ways to express the verifier duality interpretation in English are given in (51), but both of these are based on more complex structures than (44), and therefore not indirect alternatives in the sense of (30).

(51) a. Which two fingernails of yours did you paint?
b. Which pair of fingernails of yours did you paint?

The account carries over to the case of dual marking on existential and universal quantifiers. The attested cases of anti-duality from Japanese (7) and (8) involve domain duality, so we start with that. But as with *which* above, English requires a partitive structure to express domain duality, and no indirect alternative exists that could express domain duality. The best candidates below are more complex than their number-general counterparts.

(52) Taro broke one / each of the two arms.

Verifier duality, on the other hand, is actually observed with existential *one* or *a* in English. But in this case, the English numeral *two* and the plural form are generated as direct alternatives, which is fully consistent with our proposal. Finally, the type of meaning that would predict verifier anti-duality for *each* is hard to express at all in English. The very unnatural (53-a) is our best attempt. (53-b) shows that (53-b) should presuppose that Taro broke two fingers, but furthermore expresses that he didn't break each finger.

(53) a. #Taro broke each finger other than the two he did.
b. Taro didn't break each finger other than the two he did.

In sum, we have shown that our account predicts correctly that despite the presence of dual-marked quantifiers other than *both* in Icelandic and Japanese, there are no anti-duality inferences with quantifiers other than *all* and *every* in English. Such a prediction is only possible given the need for indirect alternatives to license competition with conceptual dual expressions.

Beyond the case of quantifiers, the predictions of our condition (30) are testable. We cannot exhaustively discuss this here, but only note that also for plural pronouns no anti-duality is predicted in languages that do not have dual pronouns. This prediction is correct as illustrated by (54).

(54) They, her parents, are visiting soon.

6 Alternative explanations

In the previous section, we proposed a solution for the anti-duality of *tous* using the novel notion of indirect alternative. In this section, we present possible proposals for

alternative solutions that may appear to the reader simpler, but we ultimately deem them less theoretically desirable than the indirect alternative solution.

In section 6.1, we entertain a solution in which BOTH is the core concept, instead of just DUAL. In 6.2, we entertain the possibility that *les deux NP* is directly generated as an alternative, which means that *les trois NP* also is, and so on, and find another way of blocking competition with *les n NP* for $n > 2$.

6.1 Not DUAL but BOTH is the core concept

Instead of considering DUAL as the core concept underlying *tous*'s anti-duality, we might consider BOTH instead. One advantage of this alternative proposal is that many facts fall out immediately. First, it directly explains why French *tous* is anti-dual. We can simply assume that any expression can compete with one in which a lexical item has been replaced with a core concept of the same semantic type. So *tous* is replaced with the non-lexicalized concept BOTH, similarly to where in English *all* is replaced with lexicalized *both*, and then MP applies in a standard way. It also directly explains why universal quantifiers are anti-dual, but not other expressions. In other words, it sets apart all_{DUAL} (i.e., *both*), the dual version of *all*, from $each_{\text{DUAL}}$, $which_{\text{DUAL}}$, one_{DUAL}, the_{DUAL}. So it explains why French *tous* is anti-dual, but not *each*, *which*, *one*, *the* (in French or English).

The anti-duality observed in the Japanese and Icelandic counterparts to these expressions could be simply explained with recourse to traditional Katzirian structural alternatives feeding MP, which we expect to exist regardless of whether core concepts play a part in those expressions. The lack of anti-duality of these counterparts in French and English is due to the fact that there are no dual expressions simple enough to be Katzirian alternatives.

There would thus be no need to posit indirect alternatives, which we initially proposed to restrict anti-duality effects with other quantifiers. Note that it is nevertheless compatible to have BOTH as a core concept and require an indirect alternative to license competition with it, if we want to maintain the idea that an alternative needs phonological support.

One first point of skepticism about this solution is that both from a conceptual and an empirical point of view, the primacy of the dual concept seems to extend beyond universal quantification, as already discussed in section 2. Conceptually, there are many reasons to think that the number 2 is primitive, as it is very salient in human experience, perhaps most saliently observed in the axial symmetry of human bodies. It is not obvious, however, why *both* should be conceptually more salient than *each of the two* or *which of the two*. Empirically, duality has been observed cross-linguistically in the lexicalization of pronouns on the one hand, and in that of quantificational expressions like Japanese and Icelandic, which shows that the importance of the dual concept can be lexicalized. However, one may object to this argument in observing that *both* is lexicalized much more often than other dual quantifiers (as far as we can tell; this should be of course checked).

Another point of contention is that it might seem theoretically undesirable for BOTH to be a core concept, as it appears to be more complex from a logical point of view. Indeed, BOTH can be derived from the concepts ALL and DUAL, but ALL and DUAL cannot be obviously derived from BOTH. Since ALL and DUAL underlie operators that are otherwise needed in language (in addition to being highly salient, high frequency and highly lexicalized), it might be costly to store BOTH as an additional operator, when it instead can be so easily derived.

Finally, an empirical argument against this view that BOTH is a core concept is that

there are languages including French that allow the combination of *all* and *two* in some configurations, as we saw in section 4.5. If BOTH were a core concept, we shouldn't expect this to happen.

6.2 'The n NP' are directly generated alternatives to 'all the NP'

We now consider the possibility for the definite numeral phrase *les deux NP* to be directly generated as an alternative to *tous les NP*. We discuss what is needed to allow it to be an alternative, and how to block overgeneration from competition with *les n NP* for $n > 2$.

Under a standard Katzirian notion of alternatives [Katzir, 2007], as defined in (28), *les deux NP* cannot be generated as an alternative to *tous les NP*. We could modify the Katzirian definition to replace constituents that are not necessarily of the same syntactic category.

(55) A **category-free structural alternative** of a parse tree ϕ is a parse tree ψ obtained from ϕ by deleting constituents, or replacing constituents with lexical items, not necessarily of the same syntactic category, as long as the result is well-formed.

As a result, we can replace the universal quantifier with the definite determiner, and the definite determiner with the numeral. In this way, *les deux NP* can act as a structural alternative to *tous les NP*.

Allowing *les deux NP* ('the two NP') to be directly generated as an alternative to *tous les NP* ('all the NP') gives the right result for *tous*: we have shown in section 4.1 that if *les deux NP* is generated as an alternative to *tous les NP*, MP can apply and derive anti-duality. However, this solution overgenerates. The issue we run into is that if we admit *les deux NP* to be an alternative to *tous les NP*, there is no way of blocking *les trois NP* ('the three NP') from being one too. Therefore one incorrectly predicts *tous les NP* to be odd when the domain of individuals is known to be exactly 3. And so on for all n, predicting that *tous les NP* is only possible with domains whose size is unknown or infinite, which is empirically incorrect.

The Katzirian algorithm itself, as it is stated or in its modified form in (55), specifies no way of blocking only a subset of alternatives that are of the same syntactic complexity. In other words, it cannot both allow *les 2 NP* to act as an alternative to *tous les NP* and block *les n NP* for $n > 2$. If we are to allow all these alternatives to compete with *tous les NP*, and MP applies to all, we end up with the strange inference that the number of individuals in the restrictor of the quantifier is unknown or infinite. This inference is of course generally not attested.

One possibility is to generate these alternatives *les n NP*, and then prune all the non-salient alternatives, which might the case for $n > 2$.

It is not unexpected that 2 is more salient than 3 or any higher number. However, it is unclear why (i) 2 seems to be almost always salient and (ii) 3 seems to be never salient. For instance, the following two contexts minimally differ in the number of objects present, which seems to control for saliency. Yet only in the 2-object context is *all* infelicitous.

(56) a. *Context 1: we see 3 cups, they are empty.* All the cups are empty.
b. *Context 2: we see 2 cups, they are empty.* #All the cups are empty.

It might be that from a conceptual point of view, 2 exceeds the saliency threshold, making it almost unprunable,[6] while 3 (and all numbers above) has a low enough concep-

[6]There are some exceptions, e.g. 'do you have a pencil? all of mine are unsharpened' in a context

tual saliency that it is always pruned. The latter point is especially tenuous. Even when we try to increase the saliency of 3 in whatever way possible, which usually forces the alternative to be taken into account, an anti-trial (or anti-*n*ial) inference does not seem to be derived.

(57) a. Look, we have 3 cups here, one for each child. But they are all empty!
b. My tripod has all of its legs broken.
c. All sides of this triangle are under 2cm long.
d. All sides of this square are under 2cm long.
e. All my fingers are broken.

While these examples are all grammatical and felicitous, there does seem nevertheless to be some slight effect of the number 3 being possibly borderline. It seems however unlikely that this is due to pruning because of the lack of saliency of the number 3 in the above examples, given how salient it is. It seems more likely to be an effect of *including* a non-structurally derived alternative but contextually salient alternative (note that some of the examples above have *all* combining with a bare NP, and cannot therefore have as Katzirian alternatives 'the n NP' anyways).

Another possible solution is to stipulate a restriction on possible meanings that arise from MP, which include those of the type generated here, namely forcing the domain of the quantifier to be obligatorily unknown or infinite. We would have to propose a principle that would block application of MP in a minimal non-arbitrary way. Assuming all numerals $n > 2$ are of equal saliency, blocking any number of alternatives built from numerals means blocking all alternatives built from them. Then, we can postulate that $n = 2$ has higher saliency, and therefore we don't need to block the alternative built from it. Note that such a move is somewhat reminiscent of exhaustification operators designed to avoid contradictions. One problem with this solution is that the alternative 'the 3' difficulty has the same saliency as 'the 3^{333}', or 'the n' for any n too high to be pronounced. If there are ways around such issues, they will introduce additional stipulations, weakening the plausibility of such a solution. There is enough ground for skepticism here to explore other solutions that don't rely on as many stipulations.

7 Conclusion

In this paper, we have proposed an analysis for the long-standing puzzle observed by Chemla [2007] for the anti-duality of universal quantifier *tous* in French, arising even though French has no word for *both*.

Chemla [2007] had suggested that this alternative to *tous* may involve either (i) the complex expression *les deux* ('the two'), or (ii) a core concept which need not be realized by linguistic material. In this paper, we argue that neither of these suggestions is enough alone. Option (i) does not explain why *les trois* ('the three') (and so on) is not an alternative to *tous* as well (as noted by Chemla). Option (ii), if we follow the natural assumption that the dual is a core concept, overgenerates anti-duality in expressions that don't exhibit it.

In this paper, we proposed a solution that incorporates a bit of both suggestions. There is a core concept DUAL that plays a role in an alternative to *tous*, but competition with the dual alternative is licensed only if there is an indirect alternative, i.e., a pronounceable

in which I only have two pencils. There are therefore some conditions in which the dual is prunable; describing them is beyond the scope of this paper.

expression in the language that is semantically equivalent to it.

This paper contributes to the debate about the existence of conceptual alternatives, which are alternatives not supported by linguistic material. The anti-duality of French *tous* is cited in Buccola et al. [2018] as a main example of a linguistic phenomenon where a conceptual alternative is needed. In this paper, we add some nuance to this claim. We show that a pure conceptual alternative, namely one that does not correspond in any way to a pronounceable expression, is blocked from competition. Instead, a conceptual alternative, which in this paper corresponds to a linguistic expression that cannot be pronounced, can play a role in competition, if, but only if, there is an expression that is equivalent in meaning and can be pronounced. This result raises the conjecture that at least some kind of overt expression is needed for pragmatic competition, at least between alternatives for MP, to arise.

8 Appendix: Implementation using Harbour's system

In this section, we embed the solution we proposed within system from Harbour [2014], namely, instead of having SG, PL and DUAL as number features, we have [±atomic] and [+minimal].

In this section we assume that a dual meaning is universally available from the interaction of the number features [+minimal] and [−atomic], proposed by Harbour [2014] to derive the meaning of the dual. The presence of these features will generate dual universal expressions very much in the same way as the dual core concept DUAL. It will be generated as a Katzirian alternative to the plural expression, and then, following the same assumptions introduced for our proposal in 4, it will be unpronounceable, and license competition with the definite numeral expression 'the two', deriving the anti-dual implicated presupposition.

Harbour [2014] Harbour [2014] argues that the dual meaning falls out along with singular and plural from the interaction of two features: [±atomic] and [±minimal], whose meaning is defined as the following:

(58) a. $[\![[+\text{atomic}]]\!] = \lambda P.\lambda x.P(x) \wedge atom(x)$
b. $[\![[+\text{minimal}]]\!] = \lambda P.\lambda x.P(x) \wedge \neg\exists y(P(y) \wedge y \sqsubset x)$

The minus version of these features, for Harbour [2014], is the negation of these concepts. Harbour [2014] proposes that these features compose with the meaning of a noun phrase by function application.

(59) $[\![[+\text{minimal}]\ ]\!]([\![[+\text{atomic}]\ ]\!][\![\text{N}]\!])$

The interaction of these two features derives singular, dual and plural meanings, as shown in the table below.

	[±atomic]	[±minimal]
singular	[+atomic]	[+minimal]
dual	[−atomic]	[+minimal]
plural	[−atomic]	[−minimal]

The derivations leading to each of these meanings are shown below:

(60) a. $[\![[+\text{minimal}]\,]\!]([\![[+\text{atomic}]\,]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge atom(x) \wedge \neg\exists y.atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **singular**

b. $[\![[+\text{minimal}]\,]\!]([\![[-\text{atomic}]\,]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **dual**

c. $[\![[-\text{minimal}]\,]\!]([\![[-\text{atomic}]\,]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **plural**

d. $[\![[-\text{minimal}]\,]\!]([\![[+\text{atomic}]\,]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge atom(x) \wedge \exists y.atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ contradiction

We can thus see in (60-b) how combining [−atomic] and [+minimal], we get a number specification of 2.

Updated assumptions: Trivial semantics and exhaustification We depart from Harbour [2014] in assuming that the semantics of the minus version of the features is trivial.

(61) a. $[\![\ [+\text{atomic}]\ ]\!] = \lambda P.\lambda x.P(x) \wedge atom(x)$
b. $[\![\ [-\text{atomic}]\ ]\!] = \lambda P.\lambda x.P(x)$

(62) a. $[\![\ [+\text{minimal}]\ ]\!] = \lambda P.\lambda x.P(x) \wedge \neg\exists y(P(y) \wedge y \sqsubset x)$
b. $[\![\ [-\text{minimal}]\ ]\!] = \lambda P.\lambda x.P(x)$

If this shift to trivial semantics for the negative features is not done, the anti-duality of *tous* cannot be derived as an implicated presupposition (instead, it will be encoded into any use of *tous*, regardless of the environment in which it appears). Furthermore, it has been shown that the plural itself has implicated presuppositional properties [Sauerland et al., 2005]; these can only be achieved if the meaning of the plural is unmarked, which means that the meaning of [−atomic] and [−minimal] must be left unmarked.

We will assume that the semantic inferences observed with the minus version of the features arise through exhaustification of alternatives, which include the plus version of the features (64). Furthermore, we will allow for a version of the exhaustification operator EXH to locally apply to a property, as proposed in Mayr [2015], Sauerland and Bobaljik [2022].

(63) $[\![\text{EXH}_{Alt}]\!]^w \equiv \lambda P.\lambda x.P(x)(w) \wedge \lambda Q \in Alt.\neg Q(x)(w) \vee \forall x.P(x) \rightarrow Q(x)$

This will ensure, crucially, that when [+minimal] applies to [−atomic], the dual reading is derived. We show below how the dual and plural readings are derived (the singular is derived as before, since the meanings of the plus features are left unchanged).

(64) a. Alt([−atomic])=[+atomic]
b. Alt([−minimal])=[+minimal]

(65) a. $[\![\ \text{EXH}\ [-\text{atomic}]\ \text{nP}\ ]\!] \equiv \lambda x.[\![\text{nP}]\!](x) \wedge \neg([\![\text{nP}]\!](x) \wedge atom(x))$
$\equiv \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x)$

b. $[\![\ [+\text{minimal}]\ \text{EXH}\ [-\text{atomic}]\ \text{nP}\ ]\!]$
$\equiv \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **dual**

c. $[\![\ \text{EXH}\ [-\text{minimal}]\ \text{EXH}\ [-\text{atomic}]\ \text{nP}\ ]\!]$
$\equiv \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \neg([\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x)$
$\equiv \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **plural**

When [−minimal] combines with [+atomic], EXH derives a contradiction. Here there

are several things we can say depending on the properties of the EXH operator (obligatory or optional, innocent or not) which we leave aside because they have no consequences in our analysis. No matter what the choice of the EXH operator, we get an acceptable result (either the parse is ungrammatical because of contradiction, or it corresponds to a parse with a singular meaning, which we can block or not).

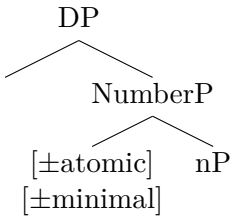
Finally, the action of the EXH operator needs to be appropriately constrained in order to capture the implicated presupposition facts. To do so, we assume that EXH is optional in non-upward-entailing environments.

The structure of a DP and universal number features Following Martí [2020], languages can be categorized into which bundles of features contrast with each other morphologically. For example, English is "[±atomic]" language, because it marks the contrast between [+atomic] (singular) and [−atomic] (plural). English contrasts with Turkish, which marks the [±minimal] contrast instead. While these two systems converge in simple cases, the difference in morphology between these two language types is observed when the noun phrases combine with numerals: in English type languages, these are plural marked, while in Turkish type languages, they are not.

We depart from Martí's system in assuming that both [±atomic] and [±minimal] features are present universally in DP structures, but that languages may not encode the featural contrasts morphologically.[7] This entails, for instance, that in English, a plural marked noun phrase will be ambiguous between a dual meaning ([−atomic] [+minimal]) and a plural meaning ([−atomic] [−minimal]), since the [±minimal] distinction is not morphologically marked. Only in very particular instances will their effect be seen. We propose that one particular instance is the anti-duality of *all*.

We assume, following Martí [2020], that a DP always has a number projection NumberP, which hosts the [±atomic] and [±minimal] features.

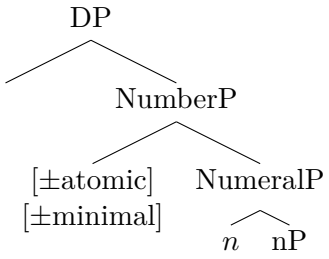
(66)



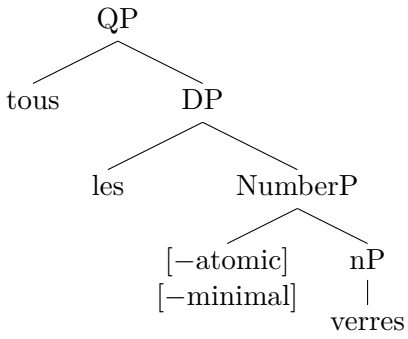
Numerals are hosted in a numeral projection, below the number projection.

[7]The universalist assumption for number features works only if the negative versions of the features have trivial semantics. This is relevant for languages like Turkish, which morphologically realize the [±minimal] features. This means that [+minimal]-marked nouns, i.e. with singular marking, will be ambiguous between having [+atomic] and [−atomic] features. If [−atomic] had contentful semantics, then we should expect singular marked nouns to be ambiguous between singular and dual meaning, which is not observed. If instead we assume that the minus version are vacuous, [−atomic] is vacuous, therefore the combination of [+minimal] and [−atomic] simply gives a singular reading. A puzzle from this remains: what if EXH applies to [−atomic] in Turkish? The combination [+minimal] EXH [−atomic] yields a dual reading, yet is morphologically marked for singular. In order to avoid this problem, we might propose that two structures that correspond to the same string cannot be alternatives to each other (here, the two strings are vacuous, corresponding to the unrealized [±atomic] features). Unless, of course, there is an indirect alternative available. This move has possibly undesirable predictions for anti-duality in Turkish, however, that we decide not to address here.

(67)



In French, a universal quantifier expression contains a definite DP. For the expression *tous les verres* ('all the cups'), we assume the following structure:



Deriving *tous*'s anti-duality Let's consider a string containing *tous*. It can correspond to several different structures, including the following two crucial ones, where (69-a) corresponds to the (unspecified) plural, and (69-b) corresponds to the dual.[8]

(69) a. tous les [−minimal] [−atomic] NP **unspecified for number**
b. tous les [+minimal] EXH [−atomic] NP **dual**

The dual expression in (69-b), after combining with a VP, is equivalent in meaning to *les deux NP VP*. We repeat its meaning in (70-a), from (65-b). By design, this semantics is equivalent to 'exactly 2', as shown in (70-b) (we use NP instead of Martí's nP for consistency).

(70) a. $[\![$ [+minimal] EXH [−atomic] NP $]\!]$
$\equiv \lambda x.[\![\text{NP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{NP}]\!](y) \wedge y \sqsubset x$
b. $\neg\exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x \equiv \forall y.[\![\text{nP}]\!](y) \wedge y \sqsubset x \rightarrow atom(y)$
$\equiv \lambda x.\{y : [\![\text{nP}]\!](y) \wedge y \sqsubseteq x \wedge atom(y)\} = 2$

[8] A string containing *tous* is in principle ambiguous between the following parses, built from [−atomic] (which morphologically marks plural) and either [+minimal] and [−minimal], and all combinations of EXH (applying non-vacuously right after − features). Since the restrictor of *tous* is DE, EXH is optional.

(68) a. (i) tous les [−minimal] [−atomic] NP **unspecified for number**
(ii) tous les [−minimal] EXH [−atomic] NP **non-singular**
(iii) tous les EXH [−minimal] [−atomic] NP **non-singular**
(iv) tous les EXH [−minimal] EXH [−atomic] NP **non-dual, non-singular**
b. (i) tous les [+minimal] [−atomic] NP **singular**
(ii) tous les [+minimal] EXH [−atomic] NP **dual**

(68-b-i) is blocked by the Avoid Ambiguity principle in (32): there is an unambiguous string 'the NP' equivalent to (68-b-i), and at most as complex. In fact, the anti-singularity inference of *tous* presumably arises via indirect competition with 'the NP' in a similar way to anti-duality. As for the parses in (68-a), they all entail the plural (and additional inferences can be derived via MP for the weaker parses). Some or all of (ii)-(iv) may be blocked, a possibility we don't address here.

We can therefore equate the meaning of this feature bundle to that of the core concept DUAL we proposed in section 4.4.1. In that section, we showed that *tous les* DUAL *NP VP* is equivalent to *les deux NP VP*. Therefore, we also have meaning equivalence between *tous les* [+minimal] EXH [−atomic] *NP VP* and *les deux NP VP*.

(71) ⟦tous les [+minimal] EXH [−atomic] NP VP⟧ ≡ ⟦tous les DUAL NP VP⟧
≡ ⟦les deux NP VP⟧

Thus, by the same Avoid Ambiguity principle from (32) that *tous les* DUAL *NP VP* was blocked, *tous les* [+minimal] EXH [−atomic] *NP VP* is also blocked. That is because it corresponds to a string that is ambiguous with another parse, and is equivalent in meaning and at most as complex as the unambiguous string *les deux NP VP*.[9]

Furthermore, we assume that *tous les* [+minimal] EXH [−atomic] *NP VP* can be generated as an alternative to *tous les* [−minimal] [−atomic] *NP VP*. This requires allowing EXH to be added into alternatives. This has been considered in the past (e.g. Spector), but really isn't ideal, due to essentially reinstating the symmetry problem that Katzir managed to at least partially address.

Thus, we derive *tous les* [+minimal] EXH [−atomic] *NP VP* as an alternative. However, since it is blocked, direct competition with it is not licensed, but *les deux NP VP* satisfies the requirements for an indirect alternative, being equivalent in meaning to *tous les* [+minimal] EXH [−atomic] *NP VP* and at most as complex as *tous les* [−minimal] [−atomic] *NP VP*, and therefore MP can apply. This derives the anti-duality of *tous les* [−minimal] [−atomic] *NP VP*.

References

Luis Alonso-Ovalle and Vincent Rouillard. Spanish bare interrogatives and number. *Journal of Semantics*, page early access, 2023. doi:10.1093/jos/ffad008.

Brian Buccola, Manuel Križ, and Emmanuel Chemla. Conceptual alternatives: Competition in language and beyond. *Paris: ENS*, 2018.

Emmanuel Chemla. French *both*: A gap in the theory of antipresupposition. *Snippets*, 15: 4–5, 2007.

Emmanuel Chemla. Presuppositions of quantified sentences: Experimental data. *Natural language semantics*, 17:299–340, 2009.

Patrick D. Elliott, Andreea C. Nicolae, and Uli Sauerland. Who and what do who and what range over cross-linguistically? *Journal of Semantics*, 39:551–579, 2022. doi:10.1093/jos/ffac008.

Daniel Harbour. Paucity, abundance, and the theory of number. *Language*, pages 185–229, 2014.

Irene Heim. On the projection problem for presuppositions. *Formal semantics–the essential readings*, pages 249–260, 1983.

[9]While this string is ambiguous between two different featural combinations [−minimal] [−atomic] and [+minimal] EXH [−atomic], the two parses are equivalent in meaning (this may actually induce blocking of one parse, but this blocking would be invisible here; see principle in X that would predict this). So the string itself is semantically unambiguous, and Avoid Ambiguity can apply.

Irene Heim. Artikel und definitheit (articles and definiteness). In A. von Stechow and D. Wunderlich, editors, *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, pages 487–535. de Gruyter, 1991.

Roni Katzir. Structurally-defined alternatives. *Linguistics and Philosophy*, 30(6): 669–690, 2007. doi:10.1007/s10988-008-9029-y. URL `https://doi.org/10.1007/s10988-008-9029-y`.

Manfred Krifka. Pragmatic strengthening in plural predications and donkey sentences. In *Semantics and linguistic theory*, volume 6, pages 136–153, 1996.

Manuel Križ. *Aspects of homogeneity in the semantics of natural language*. PhD thesis, Uniwien, 2015.

Manuel Križ and Benjamin Spector. Interpreting plural predication: Homogeneity and non-maximality. *Linguistics and Philosophy*, 44:1131–1178, 2021.

Sophia A Malamud. The meaning of plural definites: A decision-theoretic approach. *Semantics and Pragmatics*, 5:3–1, 2012.

Mora Maldonado. Plural marking and d-linking in Spanish interrogatives. *Journal of Semantics*, 37:145–170, 2020. doi:10.1093/jos/ffz024.

Luisa Martí. Numerals and the theory of number. *Semantics and Pragmatics*, 13:3–1, 2020.

Clemens Mayr. Plural definite nps presuppose multiplicity via embedded exhaustification. In *Semantics and Linguistic Theory*, volume 25, pages 204–224, 2015.

Orin Percus. Antipresuppositions. *Theoretical and empirical studies of reference and anaphora: Toward the establishment of generative grammar as an empirical science*, 52:73, 2006.

Uli Sauerland. The present tense is vacuous. *Snippets*, 6:12–13, 2002.

Uli Sauerland. Implicated presuppositions. In Anita Steube, editor, *Sentence and Context*. Mouton de Gruyter, Berlin, Germany, 2008.

Uli Sauerland and Artemis Alexiadou. Generative grammar: A meaning first approach. *Frontiers in Psychology*, 11:571295, 2020.

Uli Sauerland and Jonathan Bobaljik. Cumulative Conjunction and Exhaustification in Morphology: Clusivity, Typology, and Markedness in Person Paradigms. *unpublished*, 2022.

Uli Sauerland, Jan Anderssen, and Kazuko Yatsushiro. The plural is semantically unmarked. *Linguistic evidence: Empirical, theoretical, and computational perspectives*, pages 413–434, 2005.

Philippe Schlenker. Be articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics*, 34(3):157–212, 2008. doi:doi:10.1515/THLI.2008.013. URL `https://doi.org/10.1515/THLI.2008.013`.

Richard Sharvy. A more general theory of definite descriptions. *The philosophical review*, 89(4):607–624, 1980.

Kai von Fintel, Danny Fox, and Sabine Iatridou. Definiteness as maximal informativeness. In Luka Crnič and Uli Sauerland, editors, *The Art and Craft of Semantics: A Festschrift for Irene Heim*, pages 165–174. MIT Working Papers in Linguistics, Cambride, MA, 2014.