# Core Concepts and Indirect Alternatives: On the Anti-Duality of Quantifiers

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**Abstract**

This paper proposes an analysis for the long-standing puzzle observed by Chemla [2007] regarding the anti-duality of the French universal quantifier *tous*, which arises even though French has no word for 'both' to feed a *Maximize Presupposition* competition. This phenomenon has been cited as an example in language where a dual 'conceptual alternative' is at play [Buccola et al., 2018], but no formal account of it has been put forth. Furthermore, a naive implementation of the idea overgenerates anti-duality inferences in other expressions, such as *each*, *which* and *one* in English and French, which might be expected to be observed due to anti-dual counterparts in some languages like Icelandic and Japanese. We propose an account where French *tous* has an unpronounceable dual universal alternative built from a dual core concept, competition with which is licensed by the existence of a pronounceable expression equivalent in meaning, which we call 'Indirect Alternative'. This proposal accounts for *tous*'s anti-duality and lack of anti-$n$-ality for $n > 2$, as well as the lack of anti-duality in other quantifiers.

## 1 Introduction

The English universal quantifiers *all* and *every* are 'anti-dual', i.e., cannot be used if their domain is known to contain only two individuals. Instead, the dual universal quantifier *both* can be used in those contexts.

(1) a. #Lea broke all her arms.
b. #Lea broke every arm of hers.
c. Lea broke both her arms.

Percus [2006] and Sauerland [2008] argue that the anti-duality of *all* and *every* is due to competition with the dual universal lexical item *both*, which, via *Maximize Presupposition* [Heim, 1991, Sauerland, 2002], makes a universal quantifier with no size restriction anti-dual.

Chemla [2007] raises a puzzle for this analysis. In French, the universal quantifier *tous* is also anti-dual. But French does not have a lexical item for 'both'. In fact, the most direct translation of 'both' is the complex definite numeral expression *les deux* ('the two').

(2) a. #Léa s'est cassé tous les bras.
Léa REFL.AUX broke all the arms
#'Léa broke all her arms.'
b. Léa s'est cassé les deux bras.
Léa REFL.AUX broke the two arms
'Léa broke both her arms.'

The French data in (2) constitute a problem for the account based on *Maximize Presupposition*, for two reasons. First, *Maximize Presupposition* is generally defined for individual lexical items, but *les deux*, unlike *both*, is a complex expression (and a non-constituent string). Second, as noted by Chemla [2007], if we extend it to incorporate

complex expressions, it becomes a puzzle why *tous* can compete with *les deux*, but not with another identically structured numeral expression like *les trois* ('the three'), which would produce an unattested anti-triality inference of *tous*. Chemla therefore suggests that the explanation of *tous*'s anti-duality lies in the existence of a dual 'core concept' that can participate in competition with *tous*.[1] This observation has become one of the better-known examples suggesting the need for 'conceptual alternatives', that is, non-utterable meaningful objects that can compete with pronounceable linguistic material and be included in operations over alternatives [Buccola et al., 2018]. Nevertheless, no full-fledged account of the anti-duality of *tous* has yet been proposed. There is no discussion of what the relevant core concept is, and the lack of a formal account limits the applicability of the idea of conceptual alternatives because it leaves the predictions unspecified for structures beyond the ones considered in the original work. This paper takes on this challenge.

We aim to capture two empirical facts: a) the anti-duality of French *tous*, and its lack of anti-$n$-ality for $n > 2$, and b) the absence of anti-duality in other quantifiers, which might be expected to be observed due to anti-dual counterparts in some languages.

We propose a solution for *tous*'s anti-duality by positing the existence of universally available dual number features. In French, we propose that dual features are syncretic with plural features, and any given nominal expression will in principle be ambiguous between dual and plural. In the restrictor of the universal quantifier *tous les NP* ('all the NP'), the dual reading will therefore be generated by the grammar, but we propose that its unavailability is due to the presence of the expression *les deux NP* ('the two NP') that is equivalent in meaning and at most as complex, which blocks [tous les NP DUAL] from being pronounced due to a principle that we call Avoid Ambiguity. Following previous work, we assume that plural is number neutral, therefore the plural parse [tous les NP PL] for the string *tous les NP* is still compatible with a domain of two individuals. This is where we appeal to the familiar explanation of anti-duality using *Maximize Presupposition*, but with a twist. The plural parse [tous les NP PL] generates [tous les NP DUAL] as an alternative through familiar means. Then, [tous les NP DUAL] presupposes a domain of exactly two individuals and [tous les NP PL] presupposes nothing: by *Maximize Presupposition*, the former must be used if the domain consists of exactly two individuals. However, as mentioned above, [tous les NP DUAL] is blocked from pronunciation by Avoid Ambiguity. Is this an instance of a 'conceptual alternative' in the sense of Buccola et al. [2018]? We argue that this is too strong of a conclusion: this unpronounceable [tous les NP DUAL] is blocked only in the presence a simple enough expression equivalent in meaning, which we claim is necessary to license competition, acting as what we call an *Indirect Alternative*.

Our proposal is thus guided by the desideratum of maintaining the basic intuition, dating back to Grice, that pragmatic reasoning operates over what could have been *uttered*. However, this is technically not a necessary feature of our analysis, and we explore an alternative view without Indirect Alternatives in Jeretič et al. 2024. Ultimately, further work can determine whether Indirect Alternatives are found in other situations, and help us settle whether the case of anti-duality of *tous* involves a purely unpronounceable alternative, or relies on the presence of a pronounceable stand-in.

In section 2, we present the range of data we will aim to explain, and a summary

[1] As a reviewer points out, the term 'core concept' suggests an extra-linguistic status of the notion of duality. But duality also plays a role in grammatical agreement in many languages (see our discussion of Harbour [2014] in the appendix) and therefore has also been viewed as a syntactic feature. This gives rise to the question whether the special status of duality is rooted in grammar or outside of it. At this point, we adopt Chemla's term, but without a commitment to a primarily extra-linguistic status of duality.

of our solution. In section 3, we show that the anti-duality of French *tous* carries the signature of an implicated presupposition, corroborating Chemla's original intuition and paving the way for a solution based on competition. In section 4, we present our proposal for indirect alternatives and show how it accounts for the anti-duality of *tous*. In section 5, we discuss what our account predicts for anti-duality with other expressions cross-linguistically, including counterparts of *which, each, one, no, the, some, always*. In section 6, we entertain some plausible alternative explanations to French *tous*'s anti-duality. In section 7, we conclude.

## 2 Chemla's extended puzzle and our solution in a nutshell

Building on Chemla's (2007) observation, we present the puzzle in two parts that we aim to address. The first part concerns the special status of anti-duality with French *tous*. As Chemla discusses (also Buccola et al. 2018), there is no difficulty to use French *tous* or English *all* with a domain known to be $n$ for any $n$ greater than 2. The examples in (3) and (4) might be expected to be odd due to anti-triality and the one in (5) to anti-decality, but all are acceptable.

(3) Léa aime toutes les parties du triathlon.
Léa likes all the sections of.the triathlon
'Léa likes all sections of the triathlon.'

(4) Raphael la Tortue Ninja s'est cassé tous les doigts de sa main gauche.
Raphael the turtle ninja REFL.AUX broken all the fingers of its hand left
'Raphael the Ninja Turtle broke all the fingers on his left hand.'
*(Context: Ninja Turtles have three fingers on each hand.)*

(5) Léa s'est cassé tous les doigts.
Léa REFL.AUX broken all the fingers
'Léa broke all her fingers.'

We propose a solution that pushes forward Chemla's intuition that the number concept DUAL is special in a way that higher numbers are not. We propose that it is universally present across languages, and is the basis for an unpronounceable alternative to the universal quantifier *tous*, allowing *Maximize Presupposition* to apply, and deriving anti-duality. No anti-triality is predicted because there is no trial core concept. This part of our proposal is very similar to the one sketched by Aravind [2018, section 4.5.3, pp. 135-139] for the same puzzle. Aravind assumes the structure [DUAL all the NP], that has no spellout in French but can still act as an alternative for *Maximize Presupposition*. Some details of the analysis remain open and not straightforwardly solvable, for example how in French the spellout of [DUAL the NP] needs to be available to account for dual definite expressions, but cannot be when that same structure is combined with *all*.

The second part of our puzzle concerns cases in which anti-duality is not observed, and might be expected to arise for two reasons: (a) due to its presence in other languages; (b) given the proposal that DUAL number features are the source of the anti-duality of *tous*, it is a question of why they are not for other expressions. Indeed, Chemla's (2007) puzzle suggests that there is a parallel between the anti-duality of *all* in languages which have a lexicalized dual counterpart *both*, like English, and the anti-duality of *all* in languages that don't, like French. This parallel is not replicated for other expressions: there are a number of other items that are anti-dual in some languages, apparently due to the presence of a lexicalized dual counterpart, but are not anti-dual in other languages.

A striking example involves negative quantifiers. The English negative quantifier *no* (also *none*) is anti-dual, as shown in (6-a), which can easily be explained by *Maximize Presupposition* competition with its dual counterpart *neither*. French does not have a counterpart to *neither*, just like it does not have a counterpart to *both*; but French *aucun* ('no'/'none'), unlike *tous* ('all'), does give rise to an anti-duality inference, shown in (6-b). This is also true for German *keine* in (6-c), which does not have a word for *neither*, although it does have one for *both*.[2]

(6) a. {#None, Neither} of the sides of this sheet of paper has been used.
b. Aucun des côtés de cette feuille n'a été utilisé.
no of.the sides of this paper neg.has been used
'Neither of the sides of this sheet of paper has been used.' French
c. Keine der Seiten dieses Blattes wurde verwendet.
none.SG the.GEN sides.GEN this.GEN sheet.GEN AUX.PAST used
'Neither of the sides of this sheet of paper has been used.' German

So whatever makes *tous* anti-dual in the absence of an overt dual counterpart does not apply to *aucun*. This data suggests that anti-duality is not a general feature of a quantification expression with universal force.

This type of observation extends to other quantificational items, such as interrogative *which*, which has dual counterparts in Japanese and Icelandic, and distributive *each* and existential *one*, which have dual counterparts in Japanese. Their corresponding plural equivalents exhibit anti-duality in those languages. We show below the relevant data for *which* in Icelandic ((7); Jordan Chark (personal communication))[3] and Japanese ((8); native speaker intuition of one of the authors), and for Japanese *each* and *one* in (9) and (10) respectively.[4]

(7) a. Á hvor-um handlegg-num brotna-ði hún?
On which.DUAL-DAT arm-DAT.DEF break.INT-PST she
'Which arm did she break?' Icelandic
b. ?Á hvaða handlegg brotna-ði hún?
On which arm.DAT broke.INT-PST she

(8) a. Taroo-wa dotti-no ude-o o-tta-no?
Taro-TOP IND.DUAL-GEN arm-ACC break-past-Q
'Which arm did Taro break?' Japanese
b. #Taroo-wa dono ude-o o-tta-no?
Taroo-TOP IND arm-ACC break-past-Q

(9) a. Taroo-wa dotti-no ude-mo o-tta.
Taro-TOP IND.DUAL-GEN arm-MO break-PAST
'Taro broke each of his arms.' Japanese
b. #Taroo-wa dono ude-mo o-tta.
Taro-TOP IND arm-MO break-PAST

(10) a. Taroo-wa dotti-no ude–ka-o o-tta.
Taro-TOP IND.DUAL-GEN arm-KA-ACC break-PAST

[2]We thank our editor Yasutada Sudo (p.c.) for bringing our attention to this relevant data.

[3]For help with Icelandic data, we thank Jordan Chark, Eiríkur Rögnvaldsson, and the facebook group Málspjall (language chat).

[4]We analyze 'dotti-no … mo' as *each* because it does not allow collective or cumulative interpretations in cases where *both* and *all* do, e.g. 'Together both/all/*each of my children weighs 100 kg.'

'Taro broke one of his arms.' Japanese

b. #Taroo-wa dono ude-ka-o o-tta.
Taro-TOP IND arm-KA-ACC break-PAST

But in both English and French, no anti-duality arises in any of these cases.

(11) a. Which arm hurts you?
b. I have a problem with each arm.
c. One arm hurts me.

(12) a. Quel bras te fait mal?
which arm you cause pain
'Which arm hurts you?'
b. J'ai un problème à chaque bras.
I-have a problem to each arm
'I have a problem with each arm.'
c. Un bras me fait mal.
one arm me cause pain
'Which arm hurts you?'

On a Chemla-inspired account in which dual is a core concept, we would expect that the dual quantifiers in Japanese and Icelandic are lexicalizations of that core concept together with corresponding quantifiers. But then we would expect the possibility of that combination to occur in all languages, even those that do not lexicalize it, and generate anti-duality in the corresponding quantifiers, just like it does with French *tous*. Therefore, a proposal for the anti-duality of French *tous* should also be equipped to explain the absence of anti-duality with French and English *quel/which*, *chaque/each*, *un/one*.

As a final data point to consider, one may wonder specifically about the differences between universal quantifiers, where *all*, *every*, and *tous* are anti-dual, but not *each*, mentioned above, or definite plurals, the latter shown in (13) and (14) for English and French respectively.

(13) My arms hurt.

(14) Mes bras me font mal.
my arm me cause pain
'My arms hurt.'

An account that simply has the dual as a core concept would struggle to explain the lack of anti-duality in all of the above cases. Our proposal however provides a natural explanation for them.

In a nutshell, our solution will rely on a principle Avoid Ambiguity that blocks one parse of an ambiguous expression if there exists another expression in the language that is at most as complex and equivalent in meaning. As a result, the structure *tous les NP* DUAL is generated by the grammar but blocked due to the presence of the logically equivalent and simple enough structure *les deux NP*, while the dual versions of other expressions will not find a structure in the language with the required properties for Avoid Ambiguity to apply. For example, dual *aucun* is equivalent to *aucun des deux* or *ni l'un ni l'autre* (French expressions equivalent to 'neither'), but those expressions are too complex to block the dual parse of *aucun*. The story extends to the expressions discussed above.

The unpronounceable dual structure *tous les NP* DUAL is nevertheless generated as

an alternative to *tous les NP* PL. Thus, *Maximize Presupposition* applies and generates anti-duality. We claim that this competition is licensed because of the presence of the pronounceable expression *les deux NP*, which acts as an Indirect Alternative.

## 3 *Tous*'s anti-duality is an implicated presupposition

A necessary component of our proposal is that the anti-duality of French *tous* is indeed an implicated presupposition, i.e., generated as the result of Maximize Presupposition with another expression, and is not, for example, encoded in its lexical entry. We provide arguments for this in this section.

We first summarize the proposal of the anti-duality of English *all* as an implicated presupposition as presented in Percus [2006] and Sauerland [2008]. In particular, we will show arguments from Sauerland [2008] where anti-duality carries the signature of an implicated presupposition, namely in its epistemic status and (crucially) its projection properties under universal quantifiers, which makes it incompatible with a mere lexical specification. We also provide the third observation that the inference can be suspended in contexts in which the duality of the domain is irrelevant, which is a more general property of implicatures and implicated presuppositions. We show that all these facts hold for both English *all*/*every* and French *tous*.

The data below illustrate that the use of *all*, *every* and *tous* is odd when their domain is known to contain 1 individual, as in (15), or 2 individuals, as in (16), contrasting with a situation where the domain is known to contain more than 2 individuals, as in (17).

(15) a. #Billy broke all his noses.
b. #Billy broke every nose of his.
c. #Billy s'est cassé tous les nez.
Billy REFL.AUX broke all the noses

(16) a. #Billy broke all his legs.
b. #Billy broke every leg of his.
c. #Billy s'est cassé toutes les jambes.
Billy REFL.AUX broke all the legs

(17) a. Billy broke all his fingers.
b. Billy broke every finger of his.
c. Billy s'est cassé tous les doigts.
Billy REFL.AUX broke all the fingers

The English examples are said to be explained by the principle in (19), first proposed in Heim [1991] for indefinites and more generally by Sauerland [2002]:[5]

[5]More recent work in formal pragmatics has argued that *Maximize Presupposition* should be subsumed under grammatical exhaustification, rather than tied to speech acts as in (19) [Marty, 2017]. Some of the relevant data involve anti-duality. Specifically, Percus [2006] pointed out that (19) cannot account for anti-duality in (18).

(18) Everyone with exactly two students assigned the same exercise to both/#all of his students.

For the data we discuss in this paper, the simple account in (19) is however sufficient. We are aware of two accounts for Percus's observation in (18), namely the dynamic account by Singh [2011] and the unexhaustification account by Elliott et al. [2022b], and both accounts are compatible with the proposals made in the present paper.

(19) ***Maximize Presupposition***
Do not use $\phi$ in context $c$ if $\psi$ is an alternative to $\phi$ such that:

a. $\psi$ has a stronger presupposition than $\phi$.

b. $\phi$ and $\psi$ are contextually equivalent in $c$.

*Maximize Presupposition* (MP) states that, given two alternatives with an identical at-issue content, a speaker should not use the alternative that presupposes less. The anti-duality of *all* is generated via this principle as follows: *all* and *both* have the same at-issue content that states that a property in their scope is true of all elements in their restriction. Comparing the presuppositions of *all* and *both*, *both* presupposes that there are two individuals, while *all* presupposes nothing. Therefore, if there are two individuals in the context of which a property is true, one must use *both*, making *all* infelicitous in such a context. This explains the contrast between (16) and (17), because legs generally come in twos, but not fingers. Similarly, the anti-singularity presupposition observed in (15) is argued in Sauerland [2008] to arise from MP competition with the corresponding singular definite expression ('Billy broke his nose'), which has the same at-issue content as *all* but carries the presupposition that its domain contains only one individual.

The same mechanism applies for the anti-duality of *every*, assuming it also has the same at-issue content as *both*. Note that this story can only work if we also assume that number marking on the noun and verb are irrelevant for the possibility, since *every* calls for a singular complement, while *both* selects for a plural-marked complement.

The anti-duality of English *all*/*every* is straightforwardly explained by MP in their competition with *both*. How about French *tous*'s anti-duality? If French lacks a word for *both*, as mentioned in the introduction, we might expect its anti-duality to not arise as an implicated presupposition, but instead to be lexically encoded in the meaning of *tous*. We argue against this possibility, and show that the anti-duality of French *tous* exhibits the behavior of an implicated presupposition, in parallel with English *all*. We focus on three signature characteristics from implicated presuppositions (the first two are from Sauerland 2008): a weak epistemic status, the lack of universal projection in the scope of a universal quantifier, and its suspension when the number of individuals in the domain is irrelevant.

First, (20) shows that the anti-dual inference of *every* and *tous* is epistemically weak: if there is ignorance about whether the domain contains two individuals, these quantifiers can be used (the French data here and throughout the paper is from two native speakers).

(20) *Context: I don't know how many students there will be in my next class, there could be 2.*

a. Every student in my next class will have to work hard.

b. Dans mon prochain cours, tous mes étudiants vont devoir travailler dur.
in my next class all my students will have-to work hard

The second characteristic feature of implicated presuppositions is that they need not project universally in the scope of a universal quantifier, in contrast to a typical presupposition [Heim, 1983, Schlenker, 2008, Chemla, 2009, a.o.]. Again, we see that French *tous* patterns with English *all* in this respect:

(21) *Context: Several candidates applied. Some have written only one paper, others two, and the rest have written more than two.*

a. I checked, every candidate sent every paper of theirs.

b. J'ai vérifié, chaque candidat a envoyé tous ses articles.
I.AUX checked each candidate AUX sent all their articles

Contrast this with (22), where the dual presupposition of *both* and the uniqueness presupposition of the possessive pronoun *son* ('their') project universally:

(22) *Context: Several candidates applied. Some have written only one paper, others two, and the rest have written more than two.*

a. #Every candidate sent both papers of theirs.
⇝ *every candidate has exactly two papers*

b. #Chaque candidat a envoyé son article.
each candidate AUX sent their article
⇝ *every candidate has exactly one paper*

These projection facts rule out lexical specification of *all*'s anti-duality: if it were a presupposition (e.g., one stating the domain of *all* is not known to be 2), we should expect it to behave like one and project universally from the scope of a universal quantifier.

Finally, we show that the anti-duality inference is suspendable when the number of individuals in the domain of the quantifier is irrelevant to the question under discussion. This is expected given the general sensitivity of implicatures and implicated presuppositions to the QUD and relevance of alternatives. We show below that *all* can be used in this case even when it is known that there are two individuals in the domain.

(23) *Context: I need a pencil to write something down, and I have two pencils on my desk, but both are broken.*

a. Oh no, all my pencils are broken, I can't write.

b. Oh non, tous mes crayons sont cassés, je ne peux pas écrire.
oh no all my pencils are broken I neg can neg write

Another case that makes this point is in (24), from Lauer [2016], which works in French as well.

(24) A- Has Mary submitted the grades for all her students in her seminar?
B- How many people took it for credit?
A- [looks it up] Just two.
B- I see two grades. So **she has submitted the grades for all her students**.
B- ... Donc elle a entré les notes pour tous ses élèves.
so she has submitted the grades for all her students
(minimally changed from Lauer 2016)

In (23) and (24), an utterance with the less-presupposing form (*all*) is felicitous even though the presupposition of the stronger competitor (*both*) is entailed in the context. Intuitively, the reason is that the salient overarching QUD only 'cares about' *all* vs. *not-all* (i.e., in (24), whether professors have submitted grades to all or not all of their students), with the domain size in each particular case (i.e., whether there are two or more students) being irrelevant.

In sum, the facts above strongly suggest that the anti-dual meaning component of French *tous* is indeed an implicated presupposition.

# 4 Proposal

We have put aside the possibility that the anti-duality of French *tous* is lexically encoded and established that it carries the signature of an implicated presupposition, supporting Chemla's original proposal. This suggests that there is an alternative to *tous* that has universal semantics but presupposes duality. Once this competitor is found, the use of *tous* is predicted, via Heim's MP in (19), to implicate anti-duality.

We propose that the definite dual expression *les deux NP VP* ('the two NP VP') is an alternative to *tous les NP VP* ('all the NP VP'), albeit an atypical one, which we call an *indirect alternative*. We illustrate this novel concept in Figure 1. It is an alternative that is not generated directly by the grammar's basic alternative generation mechanism, but that is equivalent in meaning to a directly generated, but unpronounceable, alternative (written between < > in Figure 1). In this case, we posit that in French, a *tous* expression has as a directly generated alternative a dual universal expression (with meaning 'both') that is blocked from pronunciation due to the presence of the unambiguous, equivalent, and at most as complex *les deux* expression, following a principle we call Avoid Ambiguity. This will therefore allow for *les deux* to enter in competition with the universal expression, but not *les trois* ('the three') because there is no corresponding trial universal expression.

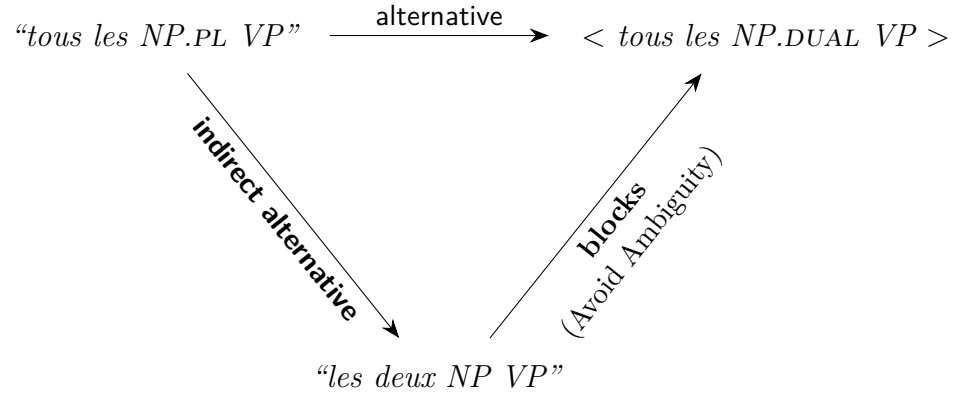



Figure 1: Indirect competition with *tous les NP VP*

Avoid Ambiguity is only applicable if there are equivalent expressions that are at most as complex. The complexity restriction will appropriately predict no anti-duality inferences for expressions other than the universal quantifier. For instance, *which of the two* is more complex than *which*, and therefore cannot block dual *which*.

We first show in section 4.1 that if *les deux NP VP* is generated as an alternative, then MP is activated, due to the at-issue meaning equivalence between *les deux NP VP* and *tous les NP VP*, and the former expression's stronger presupposition. In section 4.2, we present the core concept DUAL and how the dual universal alternative to *tous* is generated. In 4.3, we give our proposal for how competition with it is licensed, which we argue should rely on the presence of an indirect alternative. This completes our proposal.

In section 4.4, we discuss how an immediate prediction of the proposal is borne out: the possibility of saying *tous les deux* ('all the two'). In section 4.5, we discuss worries of overgeneration of our principle Avoid Ambiguity and propose a restriction of the domain in which it applies. In section 4.6, we offer a translation of this solution in a Meaning First framework, which provides a natural way to account for core concepts.

### 4.1 If 'the two' is an alternative to 'all the', then MP is licensed

In this section, we show that a sentence containing *les deux NP* and one containing *tous les NP* in the same position are contextually equivalent in a context where their presuppositions are satisfied, and only differ in their presupposed content, thus allowing for MP to apply.

(25) a. Tous les verres sont pleins.
all the cups are full
'All the cups are full.'
b. Les deux verres sont pleins.
the two cups are full
'The two cups are full.'

We start with the definite numeral expression in (25-b). Numerals are typically assumed to indicate a lower bound (e.g., 'at least 2'), and achieve an exact number reading (e.g., 'exactly 2') through exhaustification of higher-number alternatives. We will skip this bulky step, and shortcut to *deux* ('two') having the meaning 'exactly 2'. Next, we adopt a version of the familiar view of the definite article to be a maximality (MAX) operator, which presupposes existence of a maximal element, as in (26-b) [Sharvy, 1980, Link, 1983]. We take extensions of predicates to form complete join semilattices (under sum formation), see (26-c). For example, assuming a domain of two atomic cups, the denotation in (26-c) is $\{c_1, c_2, c_1 \oplus c_2\}$. The composition results in (26-d), where we obtain the unique individual $x$ that satisfies the description 'exactly two cups' and contains all subparts that satisfy the same description (this second term being superfluous in this case, just like it would for singular definites). For a domain with two atomic cups, (26-d) is equivalent to $c_1 \oplus c_2$.

(26) a. $[\![ \text{ (\textit{exactly}) deux } ]\!] = \lambda P.\lambda x.P(x) \wedge |\{y : atom(y) \wedge y \sqsubseteq x\}| = 2$
b. $[\![ \text{ les } ]\!] = \lambda P.\ \text{MAX}\big(\lambda x.P(x)\big)$[6]
c. $[\![ \text{ verres } ]\!] = \lambda x.cup(x)$
d. $[\![ \text{ les deux verres } ]\!] = \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y : atom(y) \wedge y \sqsubseteq x\}| = 2]\big)$

We now make this compose with the VP *sont pleins* ('are full'). Since *plein* ('full') is a distributive predicate, it must compose with the plural individual via a distributivity component. We introduce it as $D$ in (27-a), namely as a universal quantifier over atomic individuals (a special case of a contextual cover, which we assume the general distributivity operator to operate over, à la Schwarzschild 1996). For a domain of two individuals, either we have atomic quantification, or a collective reading (no distributivity over intermediate covers). We discuss the collective reading later.

(27) a. $[\![ D \text{ sont pleins } ]\!] = \lambda x.\forall z[[atom(z) \wedge z \sqsubseteq x] \rightarrow full(z)]$
b. $[\![ \text{ les deux verres } D \text{ sont pleins } ]\!] =$
$\forall z.\big[atom(z) \wedge z \sqsubseteq \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)\big] \rightarrow full(z)$

Now we turn to the semantics of the universal expression. We take French *tous* to

[6] $\text{MAX}(S)$ is defined iff $[\exists x.S(x) \wedge \forall z.S(z) \rightarrow z \sqsubseteq x]$; If defined, $\text{MAX}\big(S\big) = [\iota x.S(x) \wedge \forall z.S(z) \rightarrow z \sqsubseteq x]$. Note that in general lattice theory, this definition corresponds to that of a greatest element, which is different from that of a maximal element (defined as an element $x$ in $S$ such that $S(x) \wedge \neg\exists y.S(y) \wedge x < y$). However, denotations of predicates form complete join semilattices, for which greatest element and maximal element coincide.

be a universal quantifier over contextual covers of a definite plurality.[7] This semantics is based on the selectional properties of *tous*, which must compose with a definite noun phrase. We first consider a distributive reading of *tous* over atoms, given in (28-b).

(28) a. $[\![ \text{ tous } ]\!] = \lambda x.\lambda Q.\forall z.z \in C_x \rightarrow Q(z)$
$C_x$ is contextual cover of $x$, i.e. it is a contextually-supplied set of subpluralities of $x$ whose grand join equals $x$

b. Let $C_x = \{z \mid atom(z) \wedge z \sqsubseteq x\}$ (for atomic distributive predication)
$[\![ \text{ tous } ]\!] = \lambda x.\lambda Q.\forall z[[atom(z) \wedge z \sqsubseteq x] \rightarrow Q(z)]$

c. $[\![ \text{ les verres } ]\!] = \text{MAX}\big(\lambda x.cup(x)\big)$

d. $[\![ \text{ tous les verres } ]\!] = \lambda Q.\forall z[[atom(z) \wedge z \sqsubseteq \text{MAX}\big(\lambda x.cup(x)\big)] \rightarrow Q(z)]$

e. $[\![ \text{ sont pleins } ]\!] = \lambda x.full(x)$

f. $[\![ \text{ tous les verres sont pleins } ]\!] =$
$\forall z[[atom(z) \wedge z \sqsubseteq \text{MAX}\big(\lambda x.cup(x)\big)] \rightarrow full(z)]$

The at-issue meanings of the definite numeral expression in (27-b) and the *tous* expression in (28-d) are equivalent. (27-b) states that every atomic subpart of the maximal sum that makes *cup* true and has exactly 2 atoms is full. Ignoring the presuppositional component, the assertive component simply states that all cup atoms are full. This is equivalent to the meaning of (28-d). The presuppositional component of (28-d), introduced by the $\iota$ operator in the definition of MAX, is that there is a maximal sum of cups. In (27-b), it is that there is a maximal sum of cups which contains exactly two atoms. This means that if the definite numeral expression in (27-b) can be generated as an alternative to the *tous* expression in (28-d), MP applies: (27-b) and (28-d) are contextually equivalent when presuppositions are satisfied, and (27-b) has a stronger presupposition than (28-d). Therefore, if the number of atoms is known to be exactly two, the presupposition of the definite numeral expression is satisfied, and the presuppositionally weaker *tous* expression is blocked.

*Tous* phrases are also compatible with collective predication (like English 'all'), as in the examples below.

(29) a. Toutes les fourmis ont soulevé le piano.
all.FEM the ants have lifted the piano
'All the ants lifted the piano.'

b. Les deux fourmis ont soulevé le piano.
the two ants have lifted the piano
'The two ants lifted the piano.'

The collective reading of *tous* is also anti-dual. We take collective readings of *tous* to arise from quantifying over the trivial singleton cover containing the whole plurality.

(30) a. Let $C_x = \{x\}$ (for collective predication)
$[\![ \text{ toutes } ]\!] = \lambda x.\lambda Q.\forall z \in \{x\}.Q(z) \equiv \lambda x.\lambda Q.Q(x)$

b. $[\![ \text{ toutes les fourmis } ]\!] = \lambda Q.Q(\text{MAX}\big(\lambda x.ant(x)\big))$

c. $[\![ \text{ ont soulevé le piano } ]\!] = \lambda x.lift\text{-}piano(x)$

d. $[\![ \text{ toutes les fourmis ont soulevé le piano } ]\!] = lift\text{-}piano(\text{MAX}\big(\lambda x.ant(x)\big))$

This reading competes with the collective reading of the definite numeral:

[7] The orthographically singular, but phonologically identical counterpart *tout* combines with a singular definite. The semantics is the same, only the covers are over subparts of the atomic individual.

(31) a. $[\![$ les deux fourmis $]\!] = \text{MAX}\big(\lambda x.[ant(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)$
b. $[\![$ les deux fourmis ont soulevé le piano $]\!] =$
$lift\text{-}piano(\text{MAX}\big(\lambda x.[ant(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big))$

These readings are contextually equivalent, and differ only in their presuppositions, in the same way as above. The *tous* sentence presupposes the existence of a maximal sum of ants, and the definite numeral sentence presupposes the existence of a maximal sum of ants of exactly two individuals. Thus, if the context entails there are exactly two ants, and the *les deux* sentence is generated as an alternative to the *tous* sentence, it must be used, by MP. We note that there are some non-distributive predicates that are incompatible with *tous*, e.g., 'be numerous'; but the very same ones are also incompatible with the definite numeral expression (in contrast with a definite plural without a numeral). We could not find any predicate compatible with one expression but not the other.

*Tous* sentences may also involve distributivity over intermediate contextual covers; we leave working out the relevant data and details to the reader.

One detail we have so far ignored is the difference in meaning typically observed between quantificational expressions and definite plural expressions. While plural definites are interpreted as 'quasi-universal', they are associated with two properties that distinguish them from universal quantification: non-maximality, namely exception tolerance, and homogeneity, referring to an apparent wide scope reading with respect to negation [Krifka, 1996, Malamud, 2012, Križ, 2015, Križ and Spector, 2021, Bar-Lev, 2018, 2021].

(32) a. **Non-maximality:**
J'ai lu les livres sur la liste.
I read the books on the list.
$\rightsquigarrow$ *compatible with not reading one or two books on the list.*
b. **Homogeneity:**
Je n'ai pas lu les livres sur la liste.
I didn't read the books on the list.
$\rightsquigarrow$ *I read no or almost no books on the list.*

A universal quantifier which contains a plural definite, like English *all* and French *tous*, is known to erase these two properties [Link, 1983, Dowty, 1987, Brisson, 1998, Malamud, 2012, Križ, 2015]: in its presence, neither homogeneity nor non-maximality are observed.

(33) a. **Non-maximality removal by *tous/all*:**
J'ai lu tous les livres sur la liste.
I read all the books on the list.
$\rightsquigarrow$ *not compatible with not reading one or two books on the list.*
b. **Homogeneity removal by *tous/all*:**
Je n'ai pas lu tous les livres sur la liste.
I didn't read all the books on the list.
$\rightsquigarrow$ *I read some books on the list.*

In the semantics provided above, these properties are ignored. We therefore need to question whether the meaning equivalence between the *tous* expression and the definite numeral expression persists once we account for these properties. We may not need to worry: indeed, we are comparing the meaning of a *tous* expression not with a plain plural definite, but with a plural definite containing a numeral. And there are reasons to believe that a numeral may also remove homogeneity and non-maximality. Definites

numeral expressions are as of yet not well understood; as far as we know, the most detailed discussions of them can be found in Križ [2015] and Haslinger [2025]. Križ reports that the presence of a numeral appears to remove non-maximality, as shown in (34-a), but he tentatively claims that it does not remove homogeneity in English. However, he reports a possible contrast with French, where definite numerals do not seem to conserve their homogeneity. According to our own judgments,[8] there might also be a contrast between the two languages, although the data is very subtle, and it seems like there is at least one parse in both languages compatible with homogeneity removal (with a possible but yet unclear role of prosody).

(34) a. **Non-maximality removal by a numeral:**
J'ai lu les deux/dix livres sur la liste.
I read the two/ten books on the list.
⇝ *not compatible with not reading one or two books on the list.*

b. **Homogeneity removal by a numeral?**
Je n'ai pas lu les deux/dix livres sur la liste. ✓J'en ai lu que un/cinq.
I didn't read the two/ten books on the list. ?✓I only read one/five.
⇝ *compatible with reading some books on the list.*

The facts with non-maximality removal appear to be quite robust, and should be enough to make the claim that a definite numeral expression is contextually equivalent to a quantified expression, when these are unembedded. As for homogeneity removal, we will rely on the French facts, which are those we need to make our point: there exists a non-homogeneous parse of the definite numeral, that can act as a competitor to *tous* for Maximize Presupposition, assuming that the non-homogeneous reading of the definite numeral is always available when *tous* is licensed. This matters in a sentence where a quantifier expression like *tous les verres* scopes below negation, as in (35-a), and still has an anti-duality inference. This inference would rest on the non-homogeneous parse of the definite numeral expression *les deux verres* scoping below negation, as in (35-b).

(35) a. Pas tous les verres sont pleins.
not all the cups are full
'Not all the cups are full.' *odd if only 2 cups*

b. Les deux verres ne sont pas pleins.
the two cups neg are neg full
'The two cups are not full.' ✓$\neg > 2$

## 4.2 The unpronounceable dual alternative to *tous*

The data on French *tous*'s anti-duality reveals that language makes the concept 'two' important in some sense. This observation corroborates data in various domains in which duality is lexicalized, but not triality, etc. For instance, *both* is lexicalized in English, but not something equivalent to 'all the 3'. Languages have dual pronouns, but many fewer have trial pronouns.[9] Thus, there is on the one hand an intuition that the number '2' is conceptually more prominent in some way than any other higher numeral, and on the other this prominence is grammaticalized, as there are reflexes in language that show this.

[8] Judgments are of the first author, native speaker of French and English, and of the third author, native speaker of French.

[9] Other numbers can be lexicalized, e.g. *thrice*, *dozen*, etc, but are much rarer than lexicalizations of the dual. Similarly, paucal is lexicalized in some languages but less frequently than dual. Our proposal relies on these concepts to not be universally encoded in language.

Following Chemla's (2007) insight, we propose that a universal dual number concept DUAL is integrated in the grammar of every language, and that this core concept is responsible for the presence of a dual universal alternative to the French universal quantifier *tous*.

### 4.2.1 The core concept DUAL

We propose that there exist *core concepts*, which are universally present in the lexicon of any language. We assume that DUAL is either a primitive core concept, or a combination of core concepts.

The intuition that dual is a core member of our conceptual inventory is not surprising, as it has a significant presence in human experience (and any higher number does not seem to have such a presence, in both absolute terms and relative to numbers higher than it). In this sense, it may be encoded as a primitive core concept. However, this is not the only option. We can follow Harbour [2014], who argues that the numerically stable categories of number morphology are the singular, dual, and plural.[10] He proposes that dual is not a primitive feature itself, but arises from the interaction of primitive number features [−atomic] and [+minimal]. Our proposal is similar to Harbour's except that it is stronger, in assuming that the dual is universally present, meaning that both [atomic] and [minimal] features are present in all languages, despite not always morphologically expressed. Their universality is thus often invisible, but is revealed in some corners of grammar, such as with the anti-duality of *tous* in French. See the appendix (section 8) for an implementation of the following proposal using Harbour's features. In this section, we simply assume that universal number features include SG, PL and DUAL.

As number features, they can combine with any NP. In many languages, this is visible by phonologically distinguished dual number features, for example Slovenian, where they appear on the NP and its agreeing verbal morphology.

(36) Računalnik-a sta pokvarjen-a (Slovenian)
computer-DUAL are.DUAL broken-DUAL
'The computers are broken.'
(only felicitous in a context in which there are two computers)

In most languages, dual morphology is not seen. We propose that these languages still have dual number features, but they are syncretic with plural features.[11]

As a consequence, in a language where dual is unpronounced, a string containing a plural-marked NP is ambiguous between a plural and a dual interpretation. This will be possible in the scope of a universal quantifier as well, allowing the structure *tous les* DUAL *NP* to be generated.

If this structure is generated by the grammar, why doesn't *tous* have a dual meaning? In the following section, we propose a principle that blocks it from pronunciation.

### 4.2.2 Blocking the dual universal expression

**Avoid Ambiguity.** We propose a principle that blocks this dual universal structure from pronunciation because of the presence of an unambiguous string equivalent in mean-

[10] He recognizes in addition forms of paucal that are associated with a less stable numerical threshold. Since we observe anti-duality but not anti-paucality in the universal quantifier, we must say that PAUCAL is not a universally encoded core concept in the same way that DUAL is. As a result, we probably need to say that frequency of lexicalization does not fully correlate with being a core concept.

[11] If we follow Harbour in the decomposition of DUAL as [−atomic] and [+minimal], this situation can be captured very naturally by saying that [±minimal] features are null.

ing to it (once it combines with a VP), and at most as complex as it: *les deux NP*. This principle, which we call 'Avoid Ambiguity', defined in (37), encodes the preference to express a given meaning using an unambiguous string (compatible with only one logical form) rather than an ambiguous string (compatible with more than one logical form). We also encode a complexity requirement in (37), where the unambiguous string must be at most as complex as the ambiguous string, which in reality might be a result of the interaction with an independent pressure favoring less complex expressions.

(37) **Avoid Ambiguity:** if a string S is ambiguous between two parses P1 and P2, and there is a string S' with a parse P1' whose meaning is semantically equivalent to P1, but no parse P2' equivalent to P2, and S' is structurally at most as complex as S, then string S cannot realize parse P1.[12]

We propose this principle is on par with pragmatic principles like Quantity [Grice, 1975], which we assume a grammaticalized version of, following Chierchia et al. [2012] and subsequent work, by a specific operator applying at each phase boundary (see section 4.5 for more discussion on this).

The string 'tous les NP' is ambiguous between a plural reading *tous les* PL *NP* and a dual reading *tous les* DUAL *NP*. *Tous les* DUAL *NP VP* is equivalent in meaning to the expression *les deux NP VP* (as we are about to show), which is not more complex than it. Therefore, according to (37), the parse *tous les* DUAL *NP VP* cannot be pronounced.

**Complexity requirement satisfied.** Avoid Ambiguity relies on the fact that the blocking expression is at most as complex as the blocked expression, where complexity is defined as number of nodes. This requirement is satisfied for *les deux NP* blocking *tous les* DUAL *NP*.

These two expressions have identical complexity, as shown in (38), with (at least) three overt terminal nodes each (each a syntactic head). Then they both merge in identical fashion with the VP. Remember that all NPs are marked by number features, which include singular, dual or plural. In these two cases, we have DUAL (technically, the number-neutral PL is also available for the string *les deux NP*; this does not affect complexity.)

(38) a. $[_{QP}\ [_{Q}$ tous $]\ [_{DP}\quad [_{D}$ les $]\quad [_{NP}$ verres.DUAL $]]]\ [_{VP}$ sont pleins$]$
b. $[_{DP}\ [_{D}$ les $]\quad [_{NumP}\ [_{Num}$ deux $]\ [_{NP}$ verres.DUAL $]]]\ [_{VP}$ sont pleins$]$

**Meaning equivalence between *tous les* dual *NP VP* and *les deux NP VP*.** We propose that the number concept DUAL has the semantics equivalent to 'exactly 2'.[13,14]

[12] We note a potential empirical problem with (37): what if we have a string S with parse P1 and P2, and a string S' with parse P1' and P3, where ⟦P1⟧ = ⟦P1'⟧, but ⟦P2⟧ and ⟦P3⟧ are independent. Such a case might be exemplified, as brought up by an anonymous reviewer, of a determinerless language which exhibits ambiguity between definite and indefinite NPs (and also does not have word for *both*): 'all NP VP' will be ambiguous between a dual and a plural reading, which might inherit definiteness from universal quantification, while 'two NP VP' will be ambiguous between a definite and an indefinite (dual) reading. In this case, the Avoid Ambiguity principle faces a conundrum: it should block P1 of S, because S' is not ambiguous between ⟦P1⟧ and ⟦P2⟧, or P1' of S', because S is not ambiguous between ⟦P1⟧ and ⟦P3⟧? We do not know what happens empirically in such a language, and thus we leave such cases for future research.

[13] The result would have been identical, in this case, if DUAL had had a lower bound semantics, as is sometimes proposed for numerals. However, we assume here an exact semantics for DUAL, in order to align it with a proposal à la Harbour, where the combination of [−atomic] and [+minimal] yields the meaning 'exactly 2'.

[14] Singular number is standardly encoded as a presupposition [Sauerland, 2003, a.o.]. We might therefore want to encode dual as a presupposition as well. In most of this paper, this choice has no effect, because

(39) a. $[\![ \text{DUAL} ]\!] = \lambda P.\lambda x.P(x) \wedge |\{y : atom(y) \wedge y \sqsubseteq x\}| = 2$
b. $[\![ \text{les DUAL verres} ]\!] = \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)$
c. $[\![ \text{tous les DUAL verres} ]\!] =$
$\lambda Q.\forall z.[z \sqsubseteq \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)] \rightarrow Q(z)$

We compose this dual universal quantifier with the predicate *sont pleins* ('are full') (whose semantics, as assumed earlier in (27-a), involves a distributivity component).

(40) $[\![ \text{tous les DUAL verres sont pleins} ]\!] =$
$\forall z.[z \sqsubseteq \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)] \rightarrow [\forall y.atom(y) \wedge y \sqsubseteq z \rightarrow full(y)]$
$\equiv \forall z.[atom(z) \wedge z \sqsubseteq \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)] \rightarrow full(z)$

We can see that this meaning is equivalent to the one generated by the definite numeral expression, which we already derived in (27-b), repeated below.

(41) $[\![ \text{les deux verres sont pleins} ]\!] =$
$\forall z.[atom(z) \wedge z \sqsubseteq \text{MAX}\big(\lambda x.[cup(x) \wedge |\{y|atom(y) \wedge y \sqsubseteq x\}| = 2]\big)] \rightarrow full(z)$

We noted in section 4.1 that the numeral definite appears to be ambiguous between a homogeneous and non-homogeneous reading under negation. At this point the reader may wonder: why is it that non-homogeneous 'les deux NP' blocks 'tous les NP.DUAL' via Avoid Ambiguity but not the other way around? There are reasons not to worry about this, having to do with the special nature of the ambiguity of 'les deux NP'. The first point to note is that this is not a case of LF ambiguity; arguments can be found in the behavior of plural definites under *only*, for instance (see discussion in Križ and Spector [2021], Haslinger [2025]). This solves our first problem, i.e., 'les deux NP' has no non-homogeneous LF for Avoid Ambiguity to block. But if the ambiguity is not a result of two LFs, another potential problem arises: numeral definites would be underspecified for homogeneity, which would mean that they are not logically equivalent to the dual quantifier expression (as strictly non-homogeneous), which would prevent Avoid Ambiguity from applying. However, in Križ [2015] and Haslinger [2025], there are arguments against the imprecision of plural definites being a result of pure underspecification (imprecision in plural definites is an effect believed to come from the same source as homogeneity). Indeed, Križ [2015] takes the truth conditions of a plural definite to be equivalent to universal quantification; further readings of it are subsumed under the notion of "sufficient truth". We therefore simply need to make Avoid Ambiguity not sensitive to that dimension of meaning.[15]

### 4.3 Licensing competition with Indirect Alternatives and generating anti-duality

We adopt the standard alternative generation mechanism proposed by Katzir [2007], where alternatives are obtained from deleting constituents, or replacing a constituent with a lexical item of the same syntactic category. We cite the whole definition below (this merges definitions (18), (19), (20) and (41) of Katzir's (2007) paper).

the dual is embedded in a definite description that lifts it to the presuppositional level anyways. We therefore keep it at the assertive level with no theoretical commitment.

[15]Note that under negation, the homogeneous reading becomes the true one, at least for definite plurals without numerals. We do not know how this extends to numeral definites. However, we should not worry about this given that we assume that Avoid Ambiguity applies at DP boundaries, and therefore before negation, as discussed in upcoming section 4.5.

(42) A **structural alternative** of a parse tree $\phi$ is a parse tree $\psi$ obtained from $\phi$ by a finite series of deletions (removing edges and nodes), contractions (removing an edge and identifying its end nodes), and replacements of constituents in $\phi$ with lexical items of the same syntactic category or subtrees of $\phi$. [Katzir, 2007]

The notion 'of the same syntactic category' is crucial for our purposes: this restriction entails that *les n NP*, for any numeral $n$, is not generated as an alternative to *tous les NP* because *tous*, a quantifier, is not replaceable by *les*, a determiner, and *les* is not replaceable by a numeral. So *tous les NP* cannot have *les n NP*, for any $n$, as a structural alternative. Thus, *les deux NP* is not directly generated as an alternative by the Katzirian algorithm.

*Tous les* DUAL *NP VP* is a Katzirian alternative to *tous les* PL *NP VP*, where PL is replaced by DUAL. However, as we showed above, *tous les* DUAL *NP VP* is blocked from pronunciation. Our current theories of alternatives leave it undefined as to whether competition is still licensed in such a case. Here we have two theoretical options, the choice between which has consequences on our understanding of whether alternatives can be 'conceptual' in the sense of Buccola et al. [2018].

The first is to simply say that competition is possible whenever an alternative can be generated by the grammar, whether or not there is a downstream principle that blocks it. If we go down that road, competition with the alternative *tous les* DUAL *NP VP* is licensed, even if it is blocked from pronunciation by Avoid Ambiguity.

The second option is to say that competition is only licensed if there exists an *Indirect Alternative* available, which we define below.

(43) A pronounceable parse tree $I$ is an **indirect alternative** of a parse tree $S$ iff there is an unpronounceable alternative $S_X$ of $S$ such that:
(i) $[\![I]\!] \equiv [\![S_X]\!]$, and
(ii) $I \preccurlyeq S$ $:=$ $I$ has at most as many nodes as $S$

Both the Katzirian alternative and the indirect alternative is defined to be at most as complex, in some way, as the expression it is an alternative to. However, the notions of complexity used in the two definitions are not the same. Katzir's notion of complexity, which is directly integrated into the definition of structural alternatives, only allows comparison between parse trees that are related by the Katzirian algorithm. This means that the Katzirian complexity of an expression cannot be compared to that of an indirect alternative, which is by definition not related in a Katzirian way. Thus we appeal to a more general notion of complexity, which is number of nodes in the tree, and which is applicable to comparing any two parse trees.

This definition of indirect alternative mirrors the requirements for an expression that blocks another by Avoid Ambiguity. As a result, the expression *les deux NP VP* ('the two NP VP') counts as an indirect alternative to *tous les NP VP* via the unpronounceable alternative *tous les* DUAL *NP VP*. This is because *les deux NP VP* ('the two NP VP') is equivalent in meaning to *tous les* DUAL *NP VP* and is also at most as complex (as shown in section 4.2.2). Thus, on this view, we assume that direct competition with the unpronounceable dual conceptual alternative is impossible, but indirect competition is. MP applies and anti-duality arises.

We go for this second theoretical option, i.e., competition with an unpronounceable alternative is only licensed if there exists an indirect alternative. The reason is because one of the broader goals of this paper is to test the claim of Buccola et al. [2018] that there exist conceptual alternatives. This claim is a significant departure from the original intuition that led researchers to develop the notion of alternatives, i.e., that they count

as alternative utterances. Thus, we choose the theoretical option that is closer to this intuition, and see how far it can be maintained. This option that makes predictions that should be tested in future work. Indeed, while in the present case, the unpronounceability of the dual universal alternative is linked to the presence of an indirect alternative, this may not always be the case. An alternative generated by the grammar may be unpronounceable for reasons other than Avoid Ambiguity, and not have an indirect alternative available. It is such cases that will be able to distinguish between the two options. Future work is necessary to uncover them.

### 4.4 A borne out prediction: *tous les deux*

Based on the current assumptions, the analysis makes a prediction: the numeral *deux* ('two') can be in the restrictor of *tous*.

Recall that the dual universal alternative 'tous les DUAL NP' is blocked due to its phonological identity with a structure with different meaning, namely, 'tous les NP', and the availability of a semantically equivalent unambiguous structure, namely, 'les deux NP'.

Now, we can also have a structure 'tous les DUAL deux NP'. The spellout of this structure is unambiguous. Therefore it should not be blocked, and we thus predict the numeral to be in the restrictor of *tous*. This is what we observe, in some cases, as shown below.

(44) a. Tous les deux sont venus.
all the two are came
'Both came.'

b. {Les enfants, Ils} sont tous les deux venus.
the children they are all the two came
'{The children, they} both came.'

Note that this is mainly observed when the noun is elided, or in a floating quantifier position. When the noun is not elided, the use of numerals is highly restricted. This is the case not only for *deux* but for any numeral. For instance, sentences such as (45) are ill-formed.

(45) a. *Tous les deux enfants sont venus.
all the two children are came

b. *Tous les trois/dix enfants sont venus.
all the three/ten children are came

As pointed out by a reviewer however, a few examples of the type *tous les deux NP* –even though marked– can be found online. Crucially, such examples involve either *deux* or other numerals, and are equally marked.[16]

[16]See e.g., (46-a) which includes the numeral *deux* and (46-b) which includes the numeral *dix* 'ten'.

(46) a. Toutes les deux tours sont recouvertes d'un chatior pointu d'une forme de pyramide
all the two towers are covered of-a tented.roof pointy of-the shape of pyramid
octogonale.
octagonal
'Both towers are covered by a tented roof in the shape of an octogonal pyramid.'

b. Il existait des raisons plausibles de soupçonner tous les dix suspects de faire partie
there existed of.the reasons plausible of suspect all the ten suspects of do part
d'un groupe criminel organisé.
of-a group crime organized
'There were plausible grounds for suspecting all ten suspects of being part of an organized

Thus, the fact that *deux* is highly marked when the noun is not elided is due to an independent constraint on numerals being generally disallowed in the scope of *tous*. The reason behind this intriguing phenomenon is beyond the scope of this paper.

Note that in English, *all two* is ungrammatical (compare to *all three* which is not). This is predicted if *both* is a lexicalization of *all* and the dual concept, which blocks any other realization of it.

### 4.5 Predictions of Avoid Ambiguity and its Domain of Application

Our constraint Avoid Ambiguity is stated for the general case, and we should therefore be cautious that it does not overgenerate blocking. In this section, we discuss potential counterexamples to Avoid Ambiguity. We suggest that our proposal needs to be restricted in the domain that it applies to, but the predictions are borne out for the restricted domain. We discuss why there seem to be so few cases in which Avoid Ambiguity does apply.

Avoid Ambiguity predicts that no ambiguity should arise in a sentence $S$ if there is an alternative way of expressing one of its readings by an unambiguous sentence $I$ that is no more complex than $S$. One type of problematic example that comes to mind involves scope ambiguities as in (47) and (48). (47-a) and (48-a) are examples that exhibit a scope ambiguity and (47-b) and (48-b) are examples that express unambiguously one of the readings of (47-a) and (48-a) respectively. We do not think it is plausible that the b example is structurally more complex than the a example in both (45) and (46). Yet contrary to the prediction, (47-a) and (48-a) remain ambiguous.

(47) a. The teacher sent every book he had to some student. ($\forall \gg \exists$, $\exists \gg \forall$)
b. The teacher sent some student every book he had. ($\exists \gg \forall$)

(48) a. Someone has watered every tree. ($\exists \gg \forall$, $\forall \gg \exists$)
b. Every tree has been watered. ($\forall \gg \exists$)

Another type of relevant example is illustrated by (49) and (50). Again, it seems syntactically plausible that (49-b) and (50-b) are equally complex as (49-a) and (50-a) respectively by counting the number of nodes. But Avoid Ambiguity falsely predicts that (49-a) should carry an anti-duality inference and (50-a) should not allow the inverse scope construal.

(49) a. At the current time, which oak tree is visible? (which, which-of-two) (Yasutada Sudo, p.c.)
b. Which of the two oak trees is visible now? (which-of-two)

(50) a. An unmarried male is standing in front of every church. ($\exists \gg \forall$, $\forall \gg \exists$)
b. A different bachelor is standing in front of every church. ($\forall \gg \exists$)

There is a variety of arguments that could be made to argue that there is no exact meaning equivalence between these sentences to license Avoid Ambiguity: indeed there are plausibly information structural differences in (47) and (48). In (49) and (50), we question the exact synonymy between *now* and *at the current time*, or between *unmarried male* and *bachelor*; see further discussion on non-synonymy below.

However, we would like to propose as a general solution to such overgeneration that the requirement for Avoid Ambiguity applies not just at the end of the derivation, but

crime group.'

also at certain intermediate points in the derivation (a natural point to implement this requirement would be at each phase boundary). For instance, one such point would be at any CP level. If CP is such a level, we must also seriously consider smaller constituents argued to be phases, such as DPs. Above, we argued for the whole sentence *les deux NP VP* blocking *tous les NP* DUAL *VP*. This is because *les deux NP*, of type $e$, cannot be equivalent to *tous les NP*, of type $< et, t >$. However, we can simply type-lift the meaning of *les deux NP* from type $e$ to type $< et, t >$, which makes it equivalent in meaning to *tous les NP*. Avoid Ambiguity therefore must consider type-lifted meanings as well; this is not undesirable, as we assume as is standard that type-lifting is a freely available operation that can apply at any moment in the derivation to allow for composition to work (e.g., in a conjunction between a quantifier expression and definite description). A full proposal for the exact timing of Avoid Ambiguity is left for future work. We note that the timing of indirect alternative generation is also a question to ask, and in some cases in conjunction with the timing of Avoid Ambiguity (and may also be considered together with the timing of direct alternative generation, which is itself an open question).

A possible remaining worry is the case of floating quantifiers. Indeed, when *tous* is floated, as in (51), it maintains its anti-duality inference.

(51) Les verres sont tous pleins.
the glasses are all full
'The glasses are all full.'

We adopt an analysis of floating quantifiers à la Sportiche [1988], in which the DP in preverbal position originates low, together with the quantifier. Then, we assume that the Avoid Ambiguity requirement applies before this movement takes place.

Finally, Avoid Ambiguity also makes predictions for lexically ambiguous expressions, at least when the two interpretations belong to the same syntactic category. Namely, no synonym of one of the readings should be equally or less complex than the ambiguous expression. At first glance, this prediction does not seem to hold. For example for *fall*, one of its meanings can only be expressed alternatively as *act of falling*, which is more complex, but *autumn* is a synonym for the other meaning and morphologically not more complex. For *bat*, even both meanings have near-synonyms that are also not more complex, *club* for the hitting instrument and the biological term *chiroptera* for the flying mammals. It is established in the semantic-pragmatic literature that the content of non-logical vocabulary is either not visible or too flexible for some semantic generalizations [Gajewski, 2002, Del Pinal, 2019]. We could therefore assume that the licensing of Avoid Ambiguity is restricted to entailment relations that arise solely from the logical elements of a structure. However, we propose to keep the general statement of Avoid Ambiguity and appeal to a general condition of non-synonymy between two predicates, which includes "use-conditional" components of meaning such as register (these could be truth-conditional if they are encoded as presuppositions).[17] For instance, *male sibling* is higher register than *brother*, *freshmen* but not *first year student* is restricted to 4-year programs, *kick the bucket* might be archaic or dialectal while *die* is not. We therefore need to ensure that Avoid Ambiguity is sensitive to this broad notion of meaning equivalence.

As a consequence, we expect Avoid Ambiguity to only apply in cases in which logical vocabulary ends up equivalent. There seem to be very few candidates for this. Indeed universal quantification and definite descriptions with homogeneity and non-maximality removed might be the only case we can find at the level of the DP. This unfortunately

[17]Indeed, there are good reasons to believe that any competition in language must happen between expressions that are relevant at the exact same time.

makes the evidence for such a principle very restricted. However, this is perhaps not surprising theoretically: language may want to form in ways that it avoids ambiguity (and indeed more generally, non-synonymy) as a more general principle that may act in lexicon formation (in addition to encoding it as an operator).

A reviewer brings up a potential counterexample to Avoid Ambiguity: why isn't *the NP* DUAL blocked by *both dogs*? For instance, why is the following sentence possible?

(52) Two dogs walked into a room full of cats. The dogs immediately started barking.

We believe that *both dogs* is in fact not semantically equivalent to *the* DUAL *dogs*, despite being very close in meaning. Below is a context that shows they are not equivalent:

(53) *We are looking for the last remaining marbles that fell on the ground, we are not sure how many are left. After searching thoroughly I find two marbles in the corner and believe there are no more left.*

a. Look! I think I found the (two) remaining marbles!
b. #Look! I think I found both remaining marbles!

In this context, *both NP* is not felicitous, while *the NP*, or *the two NP*, is. This is enough to show that *the* DUAL *NP* is not semantically equivalent to *both NP*. Indeed, following our assumptions, *the NP* can have the parse *the* DUAL *NP* in this context. We also check that *the two NP* is felicitous, because it is assumed to be equivalent to *the* DUAL *NP*.

This example suggests that *both NP* presupposes knowledge of the exact quantity of 2 remaining marbles, while *the* DUAL *NP* does not. We leave a full investigation of the difference in meaning between these two expressions for future work.

We note that this difference in meaning is not a problem for the anti-duality of *all* and *every* in English, since *Maximize Presupposition* operates over contextual equivalence (and in any case, *the two* would be available as an indirect alternative to at least *all*[18]).

For our peace of mind, we can check that *all* is fine in this context (i.e., this context reveals no evidence against logical equivalence between *all the* and *the two* sentences):

(54) Look! I think I found all the remaining marbles!

The equivalent sentence in French are similarly fine, which is required to make our analysis stand. We furthermore show the expression *tous les deux* ('all the two') (see previous section 4.4) is also fine in the same context, which should drive this point home.

(55) – Où sont donc les dernières billes?
where are so the last marbles
So where are the remaining marbles?
– Regarde, je crois les avoir toutes les deux trouvées!
look I think them have all.FEM the two found
Look, I think I found the two [last ones]!

We have shown as a consequence that English *both NP* and *all the* DUAL *NP* do not have the same meaning. However, if the meaning difference is only presuppositional, they can both be contextually equivalent to *all the* PL *NP*, and therefore be appropriate alternatives to it for MP.

[18]However, the anti-duality of a distributive quantifier like *every* is expected to rely on the presence of a lexicalized *both* alternative: this is because *the two* is not obligatorily distributive and therefore not semantically equivalent to it, therefore cannot act as a blocker of duality and indirect alternative.

### 4.6 Indirect alternatives in the Meaning First approach

In this section, we maintain the idea of competing with a core concept, but switch to a Meaning First approach [Sauerland and Alexiadou, 2020], in which such objects can be naturally incorporated into the ontology.

The Meaning First approach contrasts with a standard syntax-first Y-model in postulating that language results from the compression of structured thoughts into phonological representations as shown in Figure 2, where thought structures are universal, and recoverable pieces of thought need not be phonologically realized.


Figure 2: Meaning First architecture of grammar [Sauerland and Alexiadou, 2020]

The Meaning First architecture assumes the existence of a set of primitive concepts from which conceptual representations are built and proposes furthermore that some of these primimites are universal. We can thus say that the 'core concept' DUAL is one of these universal primitives, and can freely combine with other concepts as part of the combinatorial system of conceptual objects that feeds language.

The Meaning First architecture assumes furthermore that there is a competition similar to scalar implicature computation or exhaustification at the thought level. For our present purposes, we state this mechanism in (56). Specifically, we assume that alternatives are generated at the thought level, but that they can only feed into competition mechanisms if they can be articulated by a form that is not more complex than the articulation of the present utterance.

(56) **Thought competition:** A thought $T$ will compete with an alternative thought $T'$ if there exists a compressed form (phonological form) of a thought $C(T'')$ such that $C(T'') \preccurlyeq C(T)$ (under some notion of complexity adapted to compressed forms), and $T'' \equiv T'$.

Furthermore we assume by adapting Katzir's (2007) notion of complexity to (56) that a thought structure always has as an alternative a thought structure that differs from it by a primitive concept (either added, removed, or replaced with another). Let $T$ be a thought of a universal quantificational claim over subparts of a plurality, which would be compressed into 'all the Ps Q'. $T^*$ is an alternative to $T$ where the core concept DUAL was added (via predicate modification) to its restrictor, where DUAL is a property of pluralities which counts its atoms and returns true if there are exactly 2.

(57) a. $T = \forall y.y \sqsubseteq \text{MAX}\big(\lambda x.P(x)\big) \rightarrow Q(y)$
b. $T^* = \forall y.y \sqsubseteq \text{MAX}\big(\lambda x.\text{DUAL}(x) \wedge P(x)\big) \rightarrow Q(y)$

As shown in section 4.2.1, $T^*$ is equivalent to the meaning of *les deux Ps Q* (i.e., to the thought which compresses into this phonological form). Therefore competition between $T$ and $T^*$ is licensed. Consequently, the anti-duality of *tous* is predicted.

## 5 Anti-Duality and its Absence with Other Quantifiers

In this section, we further discuss the second one of the puzzles for Chemla's (2007) proposal that we mentioned in the introduction. The puzzle is that the presence of a lexicalized dual quantifier in one language does not predict anti-duality of corresponding

non-dual quantifiers across languages, contrary to what one might expect given French anti-dual *tous*.

## 5.1 Lack of anti-duality of *which*, *some*, *each*

We first look at languages like Icelandic and Japanese, which express duality with quantifiers other than *all*—namely *some*, *which*, and *each*—where in neither English nor French express duality. We furthermore observe that the non-dual counterparts to those quantifiers in Icelandic and Japanese are anti-dual. But this anti-duality is not observed in English and French *some*, *which*, and *each*, which do not have dual counterparts. We initially discuss *which*-phrases, presenting the gist of our argument, and will return to other quantifiers later in this section.

### 5.1.1 No blocking of dual *which* phrases

Recall from Section 2 that, in both Icelandic and Japanese, the dual marked *which*-phrase must be used when the domain of the *which*-phrase has exactly two elements, shown in (58) and (59), repeated from (7) and (8). The number-general *which*-phrase in (b) can in both languages only be used if the domain of the *which*-phrase has three or more elements.

(58) Icelandic, repeated from (7)

a. Á hvor-um handlegg-num brotna-ði hún?
on which.DUAL-DAT arm-DAT.DEF break.INT-PST she
'Which arm did she break?'

b. ?Á hvaða handlegg brotna-ði hún?
on which arm.DAT broke.INT-PST she

(59) Japanese, repeated from (8)[19]

a. Taroo-wa dotti-no ude-o o-tta-no?
Taro-TOP IND.DUAL-GEN arm-NOM break-past-Q
'Which arm did Taro break?'

b. #Taroo-wa dono ude-o o-tta-no?
Taro-TOP IND arm-ACC break-past-Q

[19]Note that (59-a) has a genitive marker whereas (59-b) does not, possibly casting doubt that they are equivalent with respect to their structural complexity. Following Kuno [1973], we assume that the lack of genitive case marker with (59-b) is due to Haplology triggering deletion of the case marker. This means that *dono* in (59-b) is underlyingly *dono*-GEN and is thus just as complex as *dotti-no*.

Another comment about the Japanese data, pointed out by Yasutada Sudo (p.c.), is that *dotti* (as well as its formal variant *dotira*) do not exhibit a duality presupposition when used with directions. For example, (60) is acceptable in a situation where the road splits three-ways:

(60) Eki-wa **dotti**-desu-ka?
station-TOP which.direction-copula-Q
'Which direction is the station?'

This is presumably due to the fact that *dotti* in its direction sense is not the same lexical item as *dotti* translated as plain 'which', and it simply does not have a lexicalized dual variant. Further support for a lexical separation between the directional and non-directional senses of *dotti* is that the pronominal version *dore* of *dotti* does not have a directional use. (61) can be used only to ask which building is the station, not which direction the station is.

(61) Eki-wa dore-desu-ka?
station-TOP IND-COPULA-Q

The data points in (62) and (63) show that the number-general form of *which* must be used with domains of numerosity greater than two in both Icelandic and Japanese.

(62) Icelandic

a. *Á hvor-um fingr-i brotna-ði hún?
on which.dual-dat finger-dat broke.int-pst she

b. Á hvaða fingr-i brotna-ði hún?
on which finger-dat broke.int-pst she
'Which finger did she break?'

(63) Japanese

a. #Taroo-no dotti-no yubi-ga oreta-no?
Taro-gen ind-dual-gen finger-nom broke-Q

b. Taroo-no dono yubi-ga oreta-no?
Taro-gen ind finger-nom broke-Q
'Which of Taro's fingers broke?'

The number-general *which*-phrases in English and French, on the other hand, can be used also if the domain of the *which*-phrase has two elements as shown by (64) and (65).

(64) English, repeated from (11)
Which arm hurts you?

(65) French, repeated from (12)
Quel bras te fait mal?
which arm you cause pain
'Which arm hurts you?'

The absence of anti-duality in English and French is not predicted by a simple proposal in which dual is a non-lexicalized core concept with no additional stipulations, as in the ones suggested by Chemla [2007] or developed by Aravind [2018]. If this core concept exists, we expect that the dual *which* words in Icelandic and Japanese are formed by combining this dual concept and the which operator. If this is possible in Icelandic and Japanese, this combination should also be possible in other languages like French and English that do not lexicalize dual *which*. We therefore should have in those languages unpronounceable dual alternatives which would lead to anti-duality on *which* and *each* by MP. However, this is not what we observe.

We show how the account we proposed in section 4 predicts the absence of anti-duality for English and French *which*-phrases and other relevant cases. We maintain that dual *which* is indeed an alternative in those languages, but that there is no other expression that blocks its pronounciation as it would be predicted by the Avoid Ambiguity principle in (37). In other words, we assume that dual *which* is pronounceable as a simple *which* phrase, which is ambiguous between a dual and a plural meaning.

We note that there are at least two meanings one might ascribe to the combination of the dual morpheme and a quantifier, namely, duality could apply to either the domain of a quantifier or to its verifier. We define these two interpretations of number marking on a quantifier formally in (66), where we assume that a morpheme M with an interpretation $M$ of type $\langle e, t\rangle$ is a number morpheme if $\forall x, y \in D_e \,.\, \#x = \#y \rightarrow M(x) = M(y)$.

'Which one is the station?'

(66) For a number-morpheme M occurring with a quantificational noun phrase in a structure $T$ = '[Q NP]-M S', we distinguish:

a. *domain application* of $M$: The interpretation of $T$ is equivalent to the application of $M$ to the referent of 'the NP' conjoined with the interpretation of 'Q(NP)(S)'.[20]

b. *verifier application* of $M$: The interpretation of $T$ is equivalent to the application of $M$ to the referent of 'the [NP and S]' conjoined with the interpretation of 'Q(NP)(S)'.

Both of these meanings are attested cross-linguistically. Therefore both should be considered as possible alternatives available to competition with their non-dual counterparts.

First, consider domain duality. Both Icelandic *hvor* and Japanese *dotti* exhibit domain duality. This follows from the datum in (58) and (59) as the domain of *which* is the set of two arms, but the expected answer is about a singular arm. Why do we not observe domain anti-duality with *which*-questions in English? In English, one way to express domain duality in a question is to use partitive *of* and the numeral *two* as in (68).

(68) Which of the two arms hurts you?

But (68) is more complex than (the dual reading of) (64), thus the dual reading of English *which* is not blocked from pronounciation by the Avoid Ambiguity principle in (37). The same holds for any other way of expressing a dual meaning equivalent to (68) that we can think of: 'Which of your left and right arm hurts you?', 'Does your left or right arm hurt you?' and 'Which arm of two hurts you?'. Therefore, in English (as is the case in French), we do not have any expression that can stand in for the dual reading of English *which* to block it from pronounciation and make its plural reading unambiguously anti-dual by MP as an indirect alternative.

Let us now discuss verifier duality. Verifier plurality is attested with plural marking on interrogative pronouns in several languages including English, Farsi, Spanish, Hungarian, Greek, and German [Maldonado, 2020, Elliott et al., 2022a, Alonso-Ovalle and Rouillard, 2023]. Consider the English plural *which*-phrase in (69). Domain plurality would be satisfied if the addressee has multiple fingernails. But the effect of plurality in (69) is stronger – it is interpreted as a presupposition of the question that the addressee painted a plurality of their fingernails.

(69) Which fingernails of yours did you paint?

These are also languages which obligatorily mark plurality on nominals; there are languages that have obligatory dual marking, like Slovenian. We consequently observe verifier duality on *which* phrases.

[20] A question arises as to the compositionality of this meaning: in the case of *all the* DUAL *NP*, the meaning of DUAL applies to the NP meaning. But in cases of *which* phrases and existential quantifiers, applying DUAL to the NP does not yield the right meaning. One possibility is that every quantificational phrase has a syntactically represented restrictor argument, at which DUAL can apply, whether it is pronounced or not. So a 'which' phrase has the structure in (67-a) with a choice of spellouts in (67-b) and (67-c). To obtain domain application of $M$, it appears in the restrictor, as shown below.

(67) a. Structure: Which [one cup] [of the DUAL cups] is full?
b. Spellout 1: Which (one) ~~cup~~ of the DUAL cups is full?
c. Spellout 2: Which ~~one~~ cup ~~of the DUAL cups~~ is full?

(70) Kater-a računalnik-a sta pokvarjen-a?
which-DUAL computer-DUAL are.DUAL broken-DUAL?
'Which two computers are broken?'

Although this meaning is available in Slovenian, no anti-duality is predicted in counterparts with non-dual marking languages because no indirect alternative exists. Two ways to express the verifier duality interpretation in English are given in (71), but both of these are based on more complex structures than (69), and therefore do not block the dual meaning from being pronounced.

(71) a. Which two fingernails of yours did you paint?
b. Which pair of fingernails of yours did you paint?

### 5.1.2 *Each, one*

The account carries over to the case of dual marking on existential and universal distributive quantifiers. The attested cases of anti-duality from Japanese in (72) and (73) involve domain duality, so we start with that (note that in all Japanese cases we look at, namely *which*, *each* and *one*, they are built from the dual 'indeterminate' pronoun *dotti*).

(72) Repeated from (9)
a. Taroo-wa dotti-no ude-mo o-tta.
Taro-TOP IND.DUAL-GEN arm-MO break-PAST
'Taro broke each of his arms.'
b. #Taroo-wa dono ude-mo o-tta.
Taro-TOP IND arm-MO break-PAST

(73) Repeated from (10)
a. Taroo-wa dotti-no ude-ka-o o-tta.
Taro-TOP IND.DUAL-GEN arm-KA-ACC break-PAST
'Taro broke one of his arms.'
b. #Taroo-wa dono ude-ka-o o-tta.
Taro-TOP IND arm-KA-ACC break-PAST

As with *which* above, English requires a partitive structure to express domain duality, and no simple enough expression exists that could express domain duality and block the dual readings of *one* and *each*. The best candidates below are more complex than their number-general counterparts.

(74) Taro broke one / each of the two arms.

Verifier duality, on the other hand, is actually observed with existential *one* or *a* in English. But in this case, the English numeral *two* and the plural form are generated as direct alternatives, and excluded via scalar implicature, which is fully consistent with our proposal. Finally, the type of meaning that would predict verifier (anti)-duality for *each* feels unnatural, at least in English. Verifier duality for *each finger broke* would be equivalent to 'the broken fingers are 2 and each finger broke', which entails there being 2 fingers, and is equivalent in this case to domain duality at least in unembedded cases. The most natural dual expression in English is thus 'each of the two fingers broke', and the problem is the same as with domain duality.

In sum, we have shown that our account predicts correctly that despite the presence of dual-marked quantifiers other than *both* in Icelandic and Japanese, there are no anti-

duality inferences with quantifiers other than *all* and *every* in English. Such a prediction is only possible given the need for at most as complex equivalent expressions to block the dual meanings from pronounciation and stand in for them as indirect alternatives to license MP competition.

### 5.2 (Lack of) anti-duality with *no*

English has a dual negative existential quantifier *neither*, and corresponding dual NPI *either*. Their presence seems to induce an anti-duality inference in the non-dual counterparts *no(ne)* and *any*.

(75) a. (i) {#No side(s), Neither side} of this sheet of paper has/have been used.[21]
(ii) {#None, neither} of the sides of this sheet of paper has been used.
b. A: Which of the two did you pick?
B: {#None, Neither}.
c. I didn't tell {#any, either} of our parents.
d. {#None, neither} of the two (children) spoke.

In contrast, negative quantifiers in French do not seem to carry anti-duality. The French counterparts of the *no(ne)* examples in (75) are much better, as shown in (76).

(76) a. (i) Aucun côté de cette feuille n'a été utilisé.
no side of this paper neg=has been used
'{#No, Neither} side of this sheet of paper has been used.'
(ii) Aucun des côtés de cette feuille a été utilisé.
no of.the sides of this paper has been used
'{#None, neither} of the sides of this sheet of paper has been used.'
b. A: Lequel des deux as-tu choisi?
the.which of two have-you picked
'Which of the two did you pick?'
B: J'en ai choisi aucun.
I=part have picked none
'I picked neither.'
c. Je ne l'ai dit à aucun de nos parents.
I neg it.have said to none of our parents
'I didn't tell either of our parents.'
d. Aucun des deux (enfants) a parlé.
none of.the two (children) has spoken
'Neither (lit. 'none') of the two (children) spoke.'

Note that in French, all examples are fine, but we have identified an intuition that there is pressure to specify the domain with 'aucun'. This however seems to not be specific to two individuals.

German is also of interest at this point because like French it lacks a dual negative existential, while it has a dual universal *beide* ('both') like English. Like in French, the negative indefinite in German is fully acceptable with a dual domain as (77) illustrates. This shows that the anti-duality of negative indefinites in English is due to the presence of the lexical item *neither*.

[21] We note the intriguing observation that when the domain is known to be small and/or made up of specific individuals (further work required to determine this), the singular-marked negative generalized quantifier ("no side") is markedly better than a plural marked one ("no sides" or "none of the sides").

(77) Keine der Seiten kann gewinnen.
none.SG the.GEN sides can.SG win
'Neither (lit. *no*) of the the sides can win.'

This data is explained by our analysis. Indeed, while there is a dual conceptual alternative generated by applying DUAL to the domain of the quantifier, via domain application as in (66), there is no unambiguous expression equivalent in meaning to it that is at most as complex. The smallest expressions with this meaning are *aucun des deux* ('none of the two'), or *ni l'un ni l'autre* ('neither one nor the other'), which are more complex than *aucun*.

### 5.3 No anti-duality with *the*, *some*

We have so far looked at quantifiers that have dual counterparts across languages. We can also test our predictions for other cases, namely definite descriptions and *some* phrases.

Predicting that definite descriptions are not anti-dual is an important point for our theory, as they host a similar syntactic environment to *all/tous* phrases, which contain a plural definite, and thus are expected to combine with DUAL in a similar way (in possible contrast with existential or distributive quantifiers which require the dual morpheme to combine in some non-trivial way to yield domain duality). Below are the plural and dual structures for plural-marked definites in French.

(78) a. [les PL parents]
b. [les DUAL parents]

Again, there is no unambiguous expression equivalent to (78-b) that will block that one from pronounciation and act as an indirect alternative to (78-a).

Another important case to discuss is *some*, as it could be argued to predict an anti-duality inference.[22] Indeed, with the current lexical entry for DUAL, *some NP.DUAL VP* is equivalent in meaning and as complex as the unambiguous *two NP VP*. Therefore, by Avoid Ambiguity, *two NP VP* blocks *some NP.DUAL VP* from being pronounced, and acts as an indirect alternative to *some NP.PL VP*. However, this blocking and indirect competition produces no visible effect. *Some NP.PL VP* is still compatible with a domain of two, because plural is number neutral. Furthermore, if 'two NP VP' is excluded from 'some', we obtain the meaning 'exactly one'. We argue that this exclusion is not possible, because *some NP.PL VP* is anti-singular, as we can see below.

(79) Max read some books.
Anti-singularity inference: $\exists x.book(x) \wedge \neg atom(x)$

While we could leave our explanation at that, we note that the anti-singularity inference does not arise directly from our current assumptions about number. Depending on what we do to derive this inference, this may affect our assumptions about how dual number works. So we need to make sure that the additional stipulations do not create a theoretically implausible picture.

Whether singular is encoded in the assertive component, or as a presupposition (following Sauerland 2003), anti-singularity cannot be obtained from negating the singular's inference $\exists x.book(x) \wedge atom(x)$, since that would generate a contradiction. We therefore consider the anti-singularity inference of plurals to be a result of a predicate-level version of the exhaustification operator applying locally on the plural-marked NP, as argued in

[22] We thank Yasutada Sudo p.c. for bringing this up.

Mayr [2015], before it merges with the existential quantifier. If we allow for this, we must make sure that no anti-duality arises in the same case. This exhaustification crucially applies below the existential operator, i.e., below *some* is merged. For anti-duality to arise, the dual parse must be blocked by Avoid Ambiguity. However, at that point in the derivation, *two NP* is more complex than the dual parse *NP.*DUAL, and therefore does not block it.

### 5.4 *Always*: anti-dual in English but not in French

Perhaps the most promising data that provides support for the analysis of indirect alternatives has to do with the temporal quantifier *always* and its difference in meaning in English and French.

In English, *always* seems to be anti-dual when it quantifies over individual times (Yasutada Sudo, p.c.) (albeit possibly less so than *all*).

(80) a. #She came twice to visit us, and she always brought us flowers.
b. She came twice to visit us, and both times she brought us flowers.
c. She came three times to visit us, and she always brought us flowers.

The dual version of *always* is *both times*, which at first glance appears to be more complex. However, we simply assume that *always* can be decomposed into *all* and *ways*, thus making it as complex as *both times*. This allows *both times* to act as an alternative to *always* and trigger anti-duality.

Interestingly, the French word for 'always', namely *toujours*, does not seem to carry the same anti-duality effect as its English counterpart.

(81) a. Elle est venue deux fois nous rendre visite, et elle nous a toujours apporté des fleurs.
b. Elle est venue trois fois nous rendre visite, et elle nous a toujours apporté des fleurs.

*Toujours* also has a transparent morphemic decomposition as *tous* ('all') and *jours* ('days'). However, there is no expression for *both times*, the closest being *les deux fois*. However, *les deux fois* is more complex than *toujours*. Therefore the dual meaning of *toujours* is not blocked.

There is perhaps a sense in which the anti-duality of English *always* is not as clear-cut as that of *all*. And some French speakers have some reservations about *toujours* in the above sentence (others do not). A more thorough quantitative analysis might be needed to establish a clear contrast between the two languages. In the meantime, we show an excerpt from "Césarine Dietrich" by George Sand, translated into English by Edward Stanwood, which shows a use of *toujours* that is non-anti-dual, which is not kept in the English translation.

(82) – Tranchons le mot, Dubois; votre maître est fou?
'Let us speak plainly, Dubois; your master is insane, is he not?'
– Eh bien! oui, sans doute, mais il l'a déjà été deux fois, et il
well yes no doubt but he it.has already been two times and he
a **toujours** guéri.
has **always** cured
'Well yes, undoubtedly; but he has been so twice before, and was cured.'
('always cured' in the French original)

This contrast between *always* and *toujours* is particulary telling because of their morphological makeup containing *all* and *tous*. The indirect alternative theory provides a very natural explanation for why *tous* is anti-dual but not *toujours*, and for why both *all* and *always* are.

Here we note that 'always' in German (*immer*) and Japanese (*itumo*) appear to be anti-dual, just like in English.

(83) a. #Sie hat uns zweimal besucht und uns immer Blumen mitgebracht.
she has us twice visited and us always flowers brought
b. #Kanozyo-wa watasi-tati-o ni-kai tazunete-kite, itu-mo
she-TOP I-assosiative-ACC two-times visit-came when-MO
hana-o motte-kita.
flower-ACC bring-came
'She came to visit us twice and she always brought us flowers.'

For German anti-dual *immer*, we assume that it is like English decomposable into 'all' and 'ways', but has undergone suppletion. It therefore can compete with *beide Male* ('both times') and generate anti-duality.

Similar to English *always*, *itu-mo* in Japanese seems to be anti-dual (p.c. Yasutada Sudo, although the effect is rather weak, possibly similar to English *always*). *Itu-mo* consists of two morphemes, *itu* 'when' and the suffix of universal quantificational meaning. At first glance, Japanese poses a problem because the most natural translation for *both times* is *ni-kai-to-mo*, composed of 4 morphemes (or 3, depending on how the puzzling *tomo* is decomposed), as shown in (84). Therefore, we do not expect it to block the dual reading of the bimorphemic *itu-mo*, because it is more complex.

(84) Kanozyo-wa watasi-tati-o ni-kai tazunete-kite, ni-kai-to-mo
she-TOP I-associative-ACC two-times visit-came two-times-with-FOC
hana-o motte-kita.
flower-ACC bring-came
'She came to visit us twice and she brought flowers both times.'

However, we argue that there is a simpler equivalent to the dual universal quantifier *itumo*, which is *ni-kai* 'two times', which can take on a definite meaning (it can also be indefinite[23]). *ni-ka-tomo* sounds slightly more natural than just *ni-kai*, in the example above, but we leave this for future investigation.

(85) Kanozyo-wa watasi-tati-o ni-kai tazunete-kite, ni-kai hana-o
she-TOP I-associative-ACC two-times visit-came two-times flower-ACC
motte-kita.
bring-came
'She came to visit us twice and she brought flowers both times.'

[23]This opens up a new important question: why isn't Avoid Ambiguity sensitive to the ambiguity between definite and indefinite, thus predicting the definite reading of *ni kai* to be blocked altogether. Perhaps some blocking is in fact at play, explaining its weirdness relative to *ni-kai-to-mo*. We leave that question open for now.

# 6 Alternative explanations

In the previous section, we proposed a solution for the anti-duality of *tous* using the novel notion of indirect alternative. In this section, we present possible proposals for alternative solutions that may appear to the reader simpler, but we ultimately deem them less theoretically desirable than the indirect alternative solution.

In section 6.1, we entertain a solution in which BOTH is the core concept, instead of just DUAL. In 6.2, we entertain the possibility that *les deux NP* is directly generated as an alternative, which means that *les trois NP* also is, and so on, and find another way of blocking competition with *les n NP* for $n > 2$.

## 6.1 Not DUAL but BOTH is the core concept

Instead of considering DUAL as the core concept underlying *tous*'s anti-duality, we might consider BOTH instead. One advantage of this alternative proposal is that many facts fall out immediately. First, it directly explains why French *tous* is anti-dual. We can simply assume that any expression can compete with one in which a lexical item has been replaced with a core concept of the same semantic type. So *tous* is replaced with the non-lexicalized concept BOTH, similarly to where in English *all* is replaced with lexicalized *both*, and then MP applies in a standard way. It also directly explains why universal quantifiers are anti-dual, but not other expressions. In other words, it sets apart $all_{\text{DUAL}}$ (i.e., *both*), the dual version of *all*, from $each_{\text{DUAL}}$, $which_{\text{DUAL}}$, $one_{\text{DUAL}}$, $the_{\text{DUAL}}$. So it explains why French *tous* is anti-dual, but not *each*, *which*, *one*, *the* (in French or English).

The anti-duality observed in the Japanese and Icelandic counterparts to these expressions could be simply explained with recourse to traditional Katzirian structural alternatives feeding MP, which we expect to exist regardless of whether core concepts play a part in those expressions. The lack of anti-duality of these counterparts in French and English is due to the fact that there are no dual expressions simple enough to be Katzirian alternatives.

There would thus be no need to posit indirect alternatives, which we initially proposed to restrict anti-duality effects with other quantifiers. Note that it is nevertheless compatible to have BOTH as a core concept and require an indirect alternative to license competition with it, if we want to maintain the idea that an alternative needs phonological support.

One first point of skepticism about this solution is that both from a conceptual and an empirical point of view, the primacy of the dual concept seems to extend beyond universal quantification, as already discussed in section 2. Conceptually, there are many reasons to think that the number 2 is primitive, as it is very salient in human experience, perhaps most saliently observed in the axial symmetry of human bodies. It is not obvious, however, why *both* should be conceptually more salient than *each of the two* or *which of the two*. Empirically, duality has been observed cross-linguistically in the lexicalization of pronouns on the one hand, and in that of quantificational expressions like Japanese and Icelandic, which shows that the importance of the dual concept can be lexicalized. However, one may object to this argument in observing that *both* is lexicalized much more often than other dual quantifiers (as far as we can tell; this should be of course checked).

Another point of contention is that it might seem theoretically undesirable for BOTH to be a core concept, as it appears to be more complex from a logical point of view. Indeed, BOTH can be derived from the concepts ALL and DUAL, but ALL and DUAL cannot be obviously derived from BOTH. Since ALL and DUAL underlie operators that are otherwise needed in language (in addition to being highly salient, high frequency and highly

lexicalized), it might be costly to store BOTH as an additional operator, when it instead can be so easily derived.

Finally, an empirical argument against this view that BOTH is a core concept is that there are languages including French that allow the combination of *all* and *two* in some configurations, as we saw in section 4.4. If BOTH were a core concept, we shouldn't expect this to happen.

### 6.2 'The n NP' are directly generated alternatives to 'all the NP'

We now consider the possibility for the definite numeral phrase *les deux NP* to be directly generated as an alternative to *tous les NP*. We discuss what is needed to allow it to be an alternative, and how to block overgeneration from competition with *les n NP* for $n > 2$.

Under a standard Katzirian notion of alternatives [Katzir, 2007], as defined in (42), *les deux NP* cannot be generated as an alternative to *tous les NP*. We could modify the Katzirian definition to replace constituents that are not necessarily of the same syntactic category.

(86) A **category-free structural alternative** of a parse tree $\phi$ is a parse tree $\psi$ obtained from $\phi$ by deleting constituents, or replacing constituents with lexical items, not necessarily of the same syntactic category, as long as the result is well-formed.

As a result, we can replace the universal quantifier with the definite determiner, and the definite determiner with the numeral. In this way, *les deux NP* can act as a structural alternative to *tous les NP*.

Allowing *les deux NP* ('the two NP') to be directly generated as an alternative to *tous les NP* ('all the NP') gives the right result for *tous*: we have shown in section 4.1 that if *les deux NP* is generated as an alternative to *tous les NP*, MP can apply and derive anti-duality. However, as noted in the original formulation of our puzzle, this solution overgenerates. The issue we run into is that if we admit *les deux NP* to be an alternative to *tous les NP*, there is no way of blocking *les trois NP* ('the three NP') from being one too. Therefore one incorrectly predicts *tous les NP* to be odd when the domain of individuals is known to be exactly 3. And so on for all $n$, predicting that *tous les NP* is only possible with domains whose size is unknown or infinite, which is empirically incorrect.

The Katzirian algorithm itself, as it is stated or in its modified form in (86), specifies no way of blocking only a subset of alternatives that are of the same syntactic complexity. In other words, it cannot both allow *les 2 NP* to act as an alternative to *tous les NP* and block *les n NP* for $n > 2$. If we are to allow all these alternatives to compete with *tous les NP*, and MP applies to all, we end up with the strange inference that the number of individuals in the restrictor of the quantifier is unknown or infinite. This inference is of course generally not attested.

One possibility is to generate these alternatives *les n NP*, and then prune all the non-salient alternatives, which might the case for $n > 2$.

It is not unexpected that 2 is more salient than 3 or any higher number. However, it is unclear why (i) 2 seems to be almost always salient and (ii) 3 seems to be never salient. For instance, the following two contexts minimally differ in the number of objects present, which seems to control for saliency. Yet only in the 2-object context is *all* infelicitous.

(87) a. *Context 1: we see 3 cups, they are empty.* All the cups are empty.
b. *Context 2: we see 2 cups, they are empty.* #All the cups are empty.

It might be that saliency can also be conceptually based, and 2 exceeds the saliency threshold, making it prunable only with contextual support (see section X), while 3 (and all numbers above) has a low enough conceptual saliency that it is always pruned. The latter point is especially tenuous. Even when we try to increase the saliency of 3 in whatever way possible, which usually forces the alternative to be taken into account, an anti-trial (or anti-*n*ial) inference does not seem to be derived. Below are our attempts.

(88) a. Look, we have 3 cups here, one for each child. But they are all empty!
b. My tripod has all of its legs broken.
c. All sides of this triangle are under 2cm long.
d. All sides of this square are under 2cm long.
e. All my fingers are broken.

While these examples are all grammatical and felicitous, there does seem nevertheless to be some slight effect of the number 3 being possibly borderline. It seems however unlikely that this is due to pruning because of the lack of saliency of the number 3 in the above examples, given how salient it is. It seems more likely to be an effect of *including* a non-structurally derived alternative but contextually salient alternative (note that some of the examples above have *all* combining with a bare NP, and cannot therefore have as Katzirian alternatives 'the n NP' anyways).

Another possible solution is to stipulate a restriction on possible meanings that arise from MP, which include those of the type generated here, namely forcing the domain of the quantifier to be obligatorily unknown or infinite. We would have to propose a principle that would block application of MP in a minimal non-arbitrary way. Assuming all numerals $n > 2$ are of equal saliency, blocking any number of alternatives built from numerals means blocking all alternatives built from them. Then, we can postulate that $n = 2$ has higher saliency, and therefore we don't need to block the alternative built from it. Note that such a move is somewhat reminiscent of exhaustification operators designed to avoid contradictions. One problem with this solution is that the alternative 'the 3' difficulty has the same saliency as 'the $3^{333}$', or 'the $n$' for any $n$ too high to be pronounced. If there are ways around such issues, they will introduce additional stipulations, weakening the plausibility of such a solution. There is enough ground for skepticism here to explore other solutions that don't rely on as many stipulations.

### 6.3 'Les deux' as a spellout of 'tous les DUAL'

We could imagine a solution to the anti-duality of *all* in which [tous les NP.DUAL VP] is spelled out as *les deux NP VP*. If this were the case, we could do away with Avoid Ambiguity and indirect competition, and simply treat *les deux NP VP* as a directly generated alternative.

This appears to be implausible from a morphological point of view. There are two ways we can see this being implemented. The first is that *les deux* is a similar lexicalization to English *both*. A second way would be to say that DUAL agrees with *tous* and *les*, and the spellout of tous.DUAL is *les* and that of les.DUAL is *deux*. Both solutions suffer from a problem: there need to be allomorphs corresponding to possessive pronouns that appear to alternate in a regular way: *mes deux* ('my two'), *tes deux* ('your two'), etc.

# 7 Conclusion

In this paper, we have proposed an analysis for the long-standing puzzle observed by Chemla [2007] for the anti-duality of universal quantifier *tous* in French, arising even though French has no word for *both*.

Chemla [2007] had suggested that this alternative to *tous* may involve either (i) the complex expression *les deux* ('the two'), or (ii) a core concept which need not be realized by linguistic material. In this paper, we argue that neither of these suggestions is enough alone to explain the puzzle and immediately related data. Option (i) does not explain why *les trois* ('the three') (and so on) is not an alternative to *tous* as well (as noted by Chemla). Option (ii), if we follow the natural assumption that the dual is a core concept, overgenerates anti-duality in expressions that do not exhibit it.

In this paper, we proposed a solution that incorporates a bit of both suggestions. There is a core concept DUAL that plays a role in an alternative to *tous*, but competition with the dual alternative is licensed only it can be 'replaced' by an indirect alternative, i.e., a pronounceable expression in the language that is semantically equivalent to it.

This paper contributes to the debate about the existence of conceptual alternatives, which are alternatives not supported by linguistic material. The anti-duality of French *tous* is cited in Buccola et al. [2018] as a main example of a linguistic phenomenon where a conceptual alternative is needed. In this paper, we add some nuance to this claim. We show that a pure conceptual alternative, namely one that does not correspond in any way to a pronounceable expression, is blocked from competition. Instead, a conceptual alternative, which in this paper corresponds to a linguistic expression that cannot be pronounced, can play a role in competition, if, but only if, there is an expression that is equivalent in meaning and can be pronounced. This result raises the conjecture that at least some kind of overt expression is needed for pragmatic competition, at least between alternatives for MP, to arise.

Our result has, we think, implications for the structure of grammar. Namely as discussed in 4.6, our finding is most easily captured within a view of grammar where concepts are available independently of their pronunciation such as the Meaning First architecture [Sauerland and Alexiadou, 2020], but their availability as alternatives depends on them being expressible without additional effort.

This new notion of alternative might explain further problematic cases for a standard account. As noted by a reviewer, how the typical 'not all' implicature obtained with *some* arises in French is mysterious, because the *all* expression is more complex than the *some* expression, as shown below (in English this is less of an immediate issue, since the *all* expression does not come with a definite article obligatorily; note however a difference in meaning might make it a problem for English anyways).

(89) a. Quelques étudiants sont venus.
some students have come
Some students came.

b. Tous *(les) étudiants sont venus.
all the students have come
All (the) students came.

We might be able to solve this case by proposing that there is a conceptual universal alternative that does not have a direct exponent in French, but that is equivalent in meaning to the definite plural expression (in its maximal reading), in (90).

(90) Les étudiants sont venus.
the students have come
The students came.

The definite expression is now equally complex to (89-a), and can thus act as an indirect alternative to it.

# 8 Appendix: Implementation using Harbour's system

In this section, we embed the solution we proposed within system from Harbour [2014], namely, instead of having SG, PL and DUAL as number features, we have [±atomic] and [±minimal].

In this section we assume that a dual meaning is universally available from the interaction of the number features [+minimal] and [−atomic], proposed by Harbour [2014] to derive the meaning of the dual. The presence of these features will generate dual universal expressions very much in the same way as the dual core concept DUAL. It will be generated as a Katzirian alternative to the plural expression, and then, following the same assumptions introduced for our proposal in 4, it will be unpronounceable, and license competition with the definite numeral expression 'the two', deriving the anti-dual implicated presupposition.

**Harbour [2014]** Harbour [2014] argues that the dual meaning falls out along with singular and plural from the interaction of two features: [±atomic] and [±minimal], whose meaning is defined as the following:

(91) a. $[\![[+\text{atomic}]]\!] = \lambda P.\lambda x.P(x) \wedge atom(x)$
b. $[\![[+\text{minimal}]]\!] = \lambda P.\lambda x.P(x) \wedge \neg\exists y(P(y) \wedge y \sqsubset x)$

The minus version of these features, for Harbour [2014], is the negation of these concepts. Harbour [2014] proposes that these features compose with the meaning of a noun phrase by function application.

(92) $[\![[+\text{minimal}]\ ]\!]([\![[+\text{atomic}]\ ]\!][\![\text{N}]\!])$

The interaction of these two features derives singular, dual and plural meanings, as shown in the table below.

| | [±atomic] | [±minimal] |
|---|---|---|
| singular | [+atomic] | [+minimal] |
| dual | [−atomic] | [+minimal] |
| plural | [−atomic] | [−minimal] |

The derivations leading to each of these meanings are shown below:

(93) a. $[\![[+\text{minimal}]\ ]\!]([\![[+\text{atomic}]\ ]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge atom(x) \wedge \neg\exists y.atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **singular**
b. $[\![[+\text{minimal}]\ ]\!]([\![[-\text{atomic}]\ ]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **dual**
c. $[\![[-\text{minimal}]\ ]\!]([\![[-\text{atomic}]\ ]\!]([\![\text{nP}]\!]))$
$= \lambda x.[\![\text{nP}]\!](x) \wedge \neg atom(x) \wedge \exists y.\neg atom(y) \wedge [\![\text{nP}]\!](y) \wedge y \sqsubset x$ **plural**

d. ⟦[−minimal] ⟧(⟦[+atomic] ⟧(⟦nP⟧))
= $\lambda x.$⟦nP⟧$(x) \wedge atom(x) \wedge \exists y.atom(y) \wedge$ ⟦nP⟧$(y) \wedge y \sqsubset x$ contradiction

We can thus see in (93-b) how combining [−atomic] and [+minimal], we get a number specification of 2.

**Updated assumptions: Trivial semantics and exhaustification** We depart from Harbour [2014] in assuming that the semantics of the minus version of the features is trivial.

(94) a. ⟦ [+atomic] ⟧ $= \lambda P.\lambda x.P(x) \wedge atom(x)$
b. ⟦ [−atomic] ⟧ $= \lambda P.\lambda x.P(x)$

(95) a. ⟦ [+minimal] ⟧ $= \lambda P.\lambda x.P(x) \wedge \neg\exists y(P(y) \wedge y \sqsubset x)$
b. ⟦ [−minimal] ⟧ $= \lambda P.\lambda x.P(x)$

If this shift to trivial semantics for the negative features is not done, the anti-duality of *tous* cannot be derived as an implicated presupposition (instead, it will be encoded into any use of *tous*, regardless of the environment in which it appears). Furthermore, it has been shown that the plural itself has implicated presuppositional properties [Sauerland et al., 2005]; these can only be achieved if the meaning of the plural is unmarked, which means that the meaning of [−atomic] and [−minimal] must be left unmarked.

We will assume that the semantic inferences observed with the minus version of the features arise through exhaustification of alternatives, which include the plus version of the features (97). Furthermore, we will allow for a version of the exhaustification operator EXH2 to locally apply to a property, as proposed in Mayr 2015, Sauerland and Bobaljik 2022.

(96) $[\![\text{EXH2}_{Alt}]\!] \equiv \lambda P_{et}.\lambda x.P(x) \wedge \forall Q \in Alt[\neg Q(x) \vee \forall y.P(y) \rightarrow Q(y)]$

This will ensure, crucially, that when [+minimal] applies to [−atomic], the dual reading is derived. We show below how the dual and plural readings are derived (the singular is derived as before, since the meanings of the plus features are left unchanged).

(97) a. Alt([−atomic])={[+atomic],[−atomic]}
b. Alt([−minimal])={[+minimal], [−minimal]}

(98) a. ⟦ EXH [−atomic] nP ⟧ $\equiv \lambda x.$⟦nP⟧$(x) \wedge \neg($⟦nP⟧$(x) \wedge atom(x))$
$\equiv \lambda x.$⟦nP⟧$(x) \wedge \neg atom(x)$
b. ⟦ [+minimal] EXH [−atomic] nP ⟧
$\equiv \lambda x.$⟦nP⟧$(x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge$ ⟦nP⟧$(y) \wedge y \sqsubset x$ **dual**
c. ⟦ EXH [−minimal] EXH [−atomic] nP ⟧
$\equiv \lambda x.$⟦nP⟧$(x) \wedge \neg atom(x) \wedge \neg($⟦nP⟧$(x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge$ ⟦nP⟧$(y) \wedge$
$y \sqsubset x)$
$\equiv \lambda x.$⟦nP⟧$(x) \wedge \neg atom(x) \wedge \exists y.\neg atom(y) \wedge$ ⟦nP⟧$(y) \wedge y \sqsubset x$ **plural**

When [−minimal] combines with [+atomic], EXH derives a contradiction.

Finally, the action of the EXH operator needs to be appropriately constrained in order to capture the implicated presupposition facts. To do so, we assume that EXH is optional in non-upward-entailing environments.

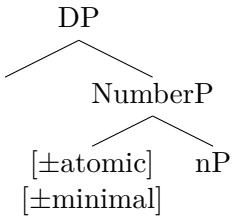
**The structure of a DP and universal number features** Following Martí [2020], languages can be categorized into which bundles of features contrast with each other

morphologically. For example, English is "[±atomic]" language, because it marks the contrast between [+atomic] (singular) and [−atomic] (plural). English contrasts with Turkish, which marks the [±minimal] contrast instead. While these two systems converge in simple cases, the difference in morphology between these two language types is observed when the noun phrases combine with numerals: in English type languages, these are plural marked, while in Turkish type languages, they are not.

We depart from Martí's system in assuming that both [±atomic] and [±minimal] features are present universally in DP structures, but that languages may not encode the featural contrasts morphologically.[24] This entails, for instance, that in English, a plural marked noun phrase will be ambiguous between a dual meaning ([−atomic] [+minimal]) and a plural meaning ([−atomic] [−minimal]), since the [±minimal] distinction is not morphologically marked. Only in very particular instances will their effect be seen. We propose that one particular instance is the anti-duality of *all*.

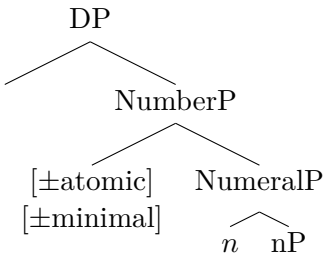
We assume, following Martí [2020], that a DP always has a number projection NumberP, which hosts the [±atomic] and [±minimal] features.

(99)



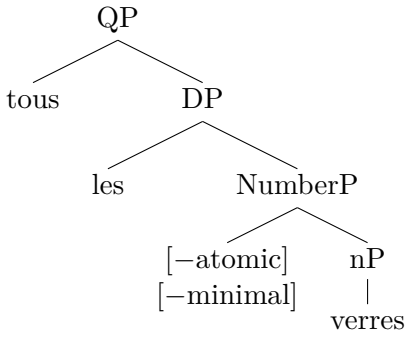

Numerals are hosted in a numeral projection, below the number projection.

(100)




In French, a universal quantifier expression contains a definite DP. For the expression *tous les verres* ('all the cups'), we assume the following structure:

[24]The universalist assumption for number features works only if the negative versions of the features have trivial semantics. This is relevant for languages like Turkish, which morphologically realize the [±minimal] features. This means that [+minimal]-marked nouns, i.e. with singular marking, will be ambiguous between having [+atomic] and [−atomic] features. If [−atomic] had contentful semantics, then we should expect singular marked nouns to be ambiguous between singular and dual meaning, which is not observed. If instead we assume that the minus version are vacuous, [−atomic] is vacuous, therefore the combination of [+minimal] and [−atomic] simply gives a singular reading. A puzzle from this remains: what if EXH applies to [−atomic] in Turkish? The combination [+minimal] EXH [−atomic] yields a dual reading, yet is morphologically marked for singular. In order to avoid this problem, we might propose that two structures that correspond to the same string cannot be alternatives to each other (here, the two strings are vacuous, corresponding to the unrealized [±atomic] features). Unless, of course, there is an indirect alternative available. This move has possibly undesirable predictions for anti-duality in Turkish, however, that we decide not to address here.




**Deriving *tous*'s anti-duality** Let's consider a string containing *tous*. It can correspond to several different structures, including the following two crucial ones, where (102-a) corresponds to the (unspecified) plural, and (102-b) corresponds to the dual.[25]

(102) a. tous les [−minimal] [−atomic] NP **unspecified for number**
b. tous les [+minimal] EXH [−atomic] NP **dual**

The dual expression in (102-b), after combining with a VP, is equivalent in meaning to *les deux NP VP*. We repeat its meaning in (103-a), from (98-b). By design, this semantics is equivalent to 'exactly 2', as shown in (103-b) (we use NP instead of Martí's nP for consistency).

(103) a. $[\![$ [+minimal] EXH [−atomic] NP $]\!]$
$\equiv \lambda x.[\![\text{NP}]\!](x) \wedge \neg atom(x) \wedge \neg\exists y.\neg atom(y) \wedge [\![\text{NP}]\!](y) \wedge y \sqsubset x$
b. $\neg\exists y.\neg atom(y) \wedge [\![\text{NP}]\!](y) \wedge y \sqsubset x \equiv \forall y.[\![\text{NP}]\!](y) \wedge y \sqsubset x \rightarrow atom(y)$
$\equiv \{y : [\![\text{NP}]\!](y) \wedge y \sqsubset x \wedge atom(y)\} = 2$

We can therefore equate the meaning of this feature bundle to that of the core concept DUAL we proposed in section 4.2.1. In that section, we showed that *tous les* DUAL *NP VP* is equivalent to *les deux NP VP*. Therefore, we also have meaning equivalence between *tous les* [+minimal] EXH [−atomic] *NP VP* and *les deux NP VP*.

(104) $[\![$tous les [+minimal] EXH [−atomic] NP VP$]\!]$ $\equiv [\![$tous les DUAL NP VP$]\!]$
$\equiv [\![$les deux NP VP$]\!]$

Thus, by the same Avoid Ambiguity principle from (37) that *tous les* DUAL *NP VP* was blocked, *tous les* [+minimal] EXH [−atomic] *NP VP* is also blocked. That is because it

[25] A string containing *tous* is in principle ambiguous between the following parses, built from [−atomic] (which morphologically marks plural) and either [+minimal] and [−minimal], and all combinations of EXH (applying non-vacuously right after − features). Since the restrictor of *tous* is DE, EXH is optional.

(101) a. (i) tous les [−minimal] [−atomic] NP **unspecified for number**
(ii) tous les [−minimal] EXH [−atomic] NP **non-singular**
(iii) tous les EXH [−minimal] [−atomic] NP **non-singular**
(iv) tous les EXH [−minimal] EXH [−atomic] NP **non-dual, non-singular**
b. (i) tous les [+minimal] [−atomic] NP **singular**
(ii) tous les [+minimal] EXH [−atomic] NP **dual**

(101-b-i) is blocked by the Avoid Ambiguity principle in (37): there is an unambiguous string 'the NP' equivalent to (101-b-i), and at most as complex. In fact, the anti-singularity inference of *tous* presumably arises via indirect competition with 'the NP' in a similar way to anti-duality. As for the parses in (101-a), they all entail the plural (and additional inferences can be derived via MP for the weaker parses). Some or all of (ii)-(iv) may be blocked, a possibility we don't address here.

corresponds to a string that is ambiguous with another parse, and is equivalent in meaning and at most as complex as the unambiguous string *les deux NP VP*.[26]

Furthermore, we assume that *tous les* [+minimal] EXH [−atomic] *NP VP* can be generated as an alternative to *tous les* [−minimal] [−atomic] *NP VP*. This requires allowing EXH to be added into alternatives, which admittedly isn't ideal.

Thus, we derive *tous les* [+minimal] EXH [−atomic] *NP VP* as an alternative. However, since it is blocked, direct competition with it is not licensed, but *les deux NP VP* satisfies the requirements for an indirect alternative, being equivalent in meaning to *tous les* [+minimal] EXH [−atomic] *NP VP* and at most as complex as *tous les* [−minimal] [−atomic] *NP VP*, and therefore MP can apply. This derives the anti-duality of *tous les* [−minimal] [−atomic] *NP VP*.

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[26] While this string is ambiguous between two different featural combinations [−minimal] [−atomic] and [+minimal] EXH [−atomic], the two parses are equivalent in meaning (this may actually induce blocking of one parse, but this blocking would be invisible here). So the string itself is semantically unambiguous, and Avoid Ambiguity in (37) can apply.

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